# How to use Slide Rules

KOLESCH & COMPANY

HOW TO USE

# SLIDE RULES.

BY

D. PETRI-PALMEDO.

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PRICE, 50 CENTS.

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# I.—PREFACE.

In preparing this booklet of instructions, the chief aim has been simplicity of expression in order to enable one, not familiar with the slide rule, to readily understand the principles upon which the same is constructed.

The simple examples given in the beginning may seem trivial to the initiated, but will, doubtlessly, be appreciated by beginners, and will enable them to more readily understand the more advanced calculations given in the latter part of the booklet.

A short, attentive study of the principles laid down herein and a little practice in solving the simple problems discussed in the following pages, will enable anyone to successfully use the slide rule in a comparatively short time.

The writer fully realizes the impossibility of having a booklet of this kind absolutely perfect. He will, therefore, be thankful for suggestions from the public and will, wherever possible, insert them in the next edition.

D. Petri-Palmedo.

*Hoboken, N. J., October, 1907.*

## II.—THE "MANNHEIM" SLIDE RULE.

The Mannheim Slide Rule is a modification of the original Gunter Scale, and has been greatly improved in recent years. It ranks nowadays as one of the most practical and time-saving mechanical contrivances, not only to the engineer and surveyor, but also to the electrician, machinist, merchant, bookkeeper, salesman, etc.

Although its principle is based on logarithms, a knowledge of the functions of the latter is not necessary to operate the rule successfully, and its merits will be discovered and appreciated after a short trial.

Multiplication and division, involution and evolution, proportions and inverted proportions, trigonometrical and logarithmic computations are solved on the rule in a purely mechanical way with the greatest ease and rapidity.

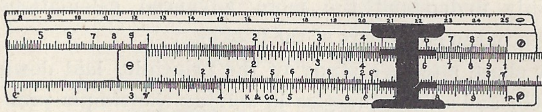


FIG. 1.

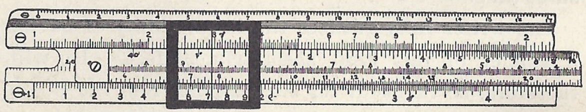


FIG. 2.

The slide rule, Figs. 1 and 2, consists of a channeled stock or body, in which is fitted a movable slide, which can be freely manipulated to the right or left or can be inverted.

The faces of the stock and of the slide are flush, and contiguous edges are fitted very accurately to facilitate close reading of the scales and to prevent any lost motion in the gliding of the slide. Upon these faces various scales are arranged longitudinally, running from left to



right, the nature of which will be explained further on. All these scales are in alignment, and an indicator sliding over the entire length of the rule serves to read off relating numbers on any two scales.

The "brass indicator," as shown in Fig. 1, is often preferred on account of its great resistance to rough handling, while the more delicate "glass indicator," shown in Fig. 2, is recommended, as more accurate readings can be obtained by means of the fine hair lines (engraved upon the glass) than by the lines engraved upon the prongs of the brass indicator. There are generally two hair lines on the glass indicator, and their distance apart on the 10″ "Precision" Slide Rule is equivalent to the ratio 1 to 12 on the upper scales. This arrangement facilitates the rapid converting of feet into inches. Two notches, one at each end of the stock, facilitate the reading of scales that will be found on the under face of the slide. A printed table of ratios and gauge-points is affixed to the back of the rule.

The main improvement, however, is the construction of the stock, or body, of the rules. The rules formerly constructed with celluloid facing on the inside of the groove only, are always liable under changes of temperature, climate or humidity, to expand, contract or warp, as shown in Figs. 3 and 4.

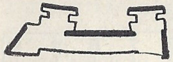


FIG. 3.

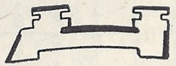


FIG. 4.

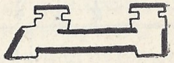


FIG. 5.

In the "Precision" Slide Rules this drawback is entirely overcome by mounting both the inner and outer face of the stock with celluloid, as shown in Fig. 5. This prevents the rule from being affected by humidity or changes in temperature. The ends of the celluloid facings on the "Precision" Slide Rules are secured to the wood by German silver screws, which secure their absolute adherence, Figs. 1 and 2.

## III.—GRAPHICAL ADDITION AND SUBTRACTION.

The operations with the slide rule depend upon what is called "graphical addition and subtraction," although addition and subtraction as such are not performed on it.

Adding and subtracting can be performed in a purely mechanical way by adding up or subtracting from each other, spaces graphically laid down and representing the arithmetical values of the respective numbers.

**Graphical Addition.**—If of two ordinary scales divided into equal parts each, the digit 0 on the upper scale is brought in coincidence with the 3 on the bottom scale, as shown in Fig. 6, then below every number on the top scale will be found the sum of this number +3.

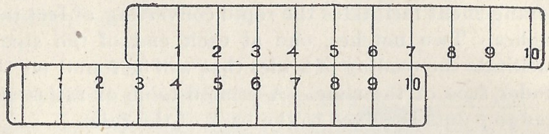



FIG. 6.

**Graphical Subtraction.**—In the same simple manner the difference of two numbers can be found. For instance, if 6 is to be deducted from 10, set 6 on bottom scale underneath 10 on top scale and above 0 on lower scale, the difference 4 will be found. See Fig. 7.

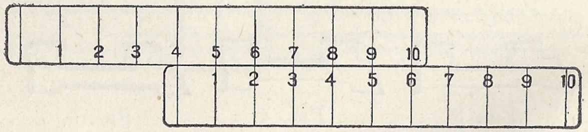



FIG. 7.

The operation of the slide rule is nothing more than graphical addition and subtraction, but the spaces of the various scales between consecutive numbers 1, 2, 3 are not equal; that is, the spaces from the beginning of the scale up to the various numbers are not proportional to the numbers themselves but to the logarithms of these numbers, and it is the graphical addition and subtraction of these unequal spaces by which the various calculations are performed on the slide rule.



## IV.—THE LOGARITHMIC SCALE.

A scale in which the length of the spaces correspond to the logarithms of consecutive numbers is called a **logarithmic scale.** This is the keynote of operations on the slide rule, and it will be well to go into the making of such a scale at length. In a table of logarithms the logarithm of 1 is zero, therefore, this figure (the left hand index) is found at the beginning of the scale. The logarithm of 10 is by a table of logarithms equal to one; therefore, in making a slide rule scale, we lay one space of a convenient length (say 5″) and mark the end of it 10 (see Fig. 8). The logarithm of 2 is equal to .301, there-

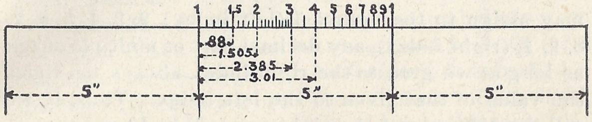



FIG. 8.

fore, we take of one unit length of 5″, .301 parts= .301×5″=1.505″ and lay this distance off from the beginning of the scale and mark the end of this distance 2. In the same manner for log. 3=.477, we lay off a distance equal to .477×5″=2.385″ and mark the end of it 3, and so forth. In the same manner the distances between the consecutive numbers are obtained. For instance, the logarithm of 1.5 is equal to .176, thus we lay off a distance equal to .176×5″= 88″. In order not to overcrowd the slide rule scale with figures, not all are numbered. They are, however, made easily distinguishable and readable by longer and shorter division lines, much in the same manner as the halves, tenths and hundredths on the ordinary decimal measuring scale.

The logarithm of 100 is equal to two. Thus, if we wish to prolong our scale beyond 10, we must add for 100 another unit length of 5″ and lay off distances for the intermediate numbers 20, 30, 40, etc , in the same manner as was done above for the numbers between 1 and 10, and in doing so we find **that the spaces for the numbers between 10 and 100 are exact duplicates of the**

**corresponding numbers between 1 and 10.** Likewise, the logarithm of 1,000 being three, we might add another similar scale 5″ long, divided up in the same manner. On the other hand, we may extend the scale beyond 1 to the left as follows: The logarithm of $\frac{1}{10}$ (.1) is equal to minus one (—1), that is, we must lay off for this number a unit space of 5″ to the left of 1 and mark the end of it .1. In laying off the logarithm of the intervening numbers, .2, .3, .4, etc , we will find that we again obtain the exact counterpart of the original scale 1-10. It is, therefore, evident that any of these scales, 1-10, or 10-100, or 100-1,000, or .1-1, or .01-.1 may be used promiscuously for one another, or, in other words, one scale will serve for all. Or, in still other words, we may assign to the figures 1 (left index), 2, 3, 4, 5, 6, 7, 8, 9, 1, (right index), any decimal part or multiple of ten as long as we give to the right index always ten times the value of that given to the left index. Thus, if we call the left index 1 the right one will be 10 and the intermediate numbers 2, 3, 4, 5, 6, 7, 8, 9. If the left index is 10 the right will be 100 and the intermediate numbers 20, 30, 40, 50, 60, 70, 80, 90. If the left index is .1 the right will be 1 and the intermediate numbers .2, .3, .4, .5, .6, .7, .8, .9. If the left index is .01 the right is .1, and so forth. There are, as inspection of the slide rule will reveal (Fig. 1), two such scales of unit length, end for end, on the upper edges of the rule and slide; each one of these scales is to be considered by itself. The middle index (1) of these double scales, which we will hereafter designate as A (rule) and B (slide), is the right index for the first scale and the left index for the second scale.

Thus, if the left index of the double scale is 1 the middle one is 10 and the right one is 100; or if left is 10, the middle one is 100 and the right 1,000; or if the left is .1, the middle is 1 and the right 10, and so forth. The lower scales on the rule, which we will hereafter call scales C and D, are in all respects the same as A and B; but the unit chosen is twice the length of that chosen for scales A and B, so that the spaces between corresponding numbers are all twice as large. This



affords an opportunity to carry the subdivisions further and makes a closer "reading" possible. The beginner should practice reading the subdivisions, for which Fig. 9 will prove helpful. He should also notice that the

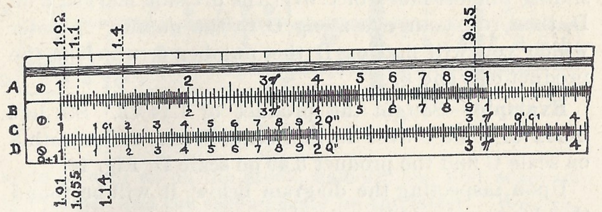



Fig. 9.

subdivisions are carried further in the large spaces than in the small. Thus, the smallest subdivided space on the upper A and B scales, between 1 and 2, is .02 ($\frac{2}{100}$) while that between 9 and 10, for instance, is only .1 ($\frac{1}{10}$). Intermediate distances must be estimated. Thus, if the hair-line of the indicator falls midway between two consecutive division lines, one would read one-half of this distance. Thus the last, for instance in Fig. 9, would read 9.35, the last figure .05 being obtained by estimating the distance between 9.3 and 9.4. A little scrutinizing of the various subdivisions will soon enable the learner to read correctly and rapidly. We are now ready to proceed with the use of the slide rule, and will take up the various operations one by one, stating first the underlying principle and giving next the mode of operation, with illustrating examples.

## V.—MULTIPLICATION.

**Principle.**—If the logarithms of two numbers are added together, their sum corresponds to the product of these two numbers.

**Mode of Operation.**—The adjacent lower scales, C and D, are preferably employed for calculations, as, on account of their greater length, they are subdivided more finely, and consequently yield more accurate readings than the two upper scales, which are only one-half their length. Based on the principle stated above, multiplication is performed by graphically adding the logarithmic spaces representing the numerical values of the respective factors. Test this by adding the space marked 2 on scale C to the space marked 3 on scale D by sliding the left index of C over the division marked 3 on D, then run your eye along C to the number 2, under which you will find on D the number 6, which is the product of 2 and 3.

**Example.**—Sought the product of 1.14×3. Set left index of scale C over 1.14 on scale D and underneath 3 on scale C find the product 3.42 on scale D, Fig. 10.

Upon inspecting the diagram below, it will be found that by merely adding up the spaces representing 1.14 and 3 the product 3.42 of these two factors is obtained.

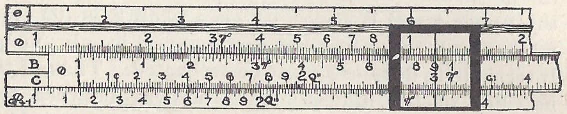



FIG. 10.

The reading will be found more readily when taken by means of the indicator.

In the two examples given, there was no difficulty in reading the result on the lower scale D. But now try and multiply the numbers 4 and 5, for instance. Slide the left index of scale C over the 4 mark of scale D and try to read the product on the scale D under the 5 mark of scale C, and you will find it to be in the air, that is, beyond the length of the rule. To overcome this difficulty, we will turn to the upper scales A and B for a while. Repeat the operation by sliding the left index of the scale B under the number 4 of scale A, as shown in Fig. 11, and running your eye along scale B you will find over 5 of the B scale the product 4×5=20 on the second half of the A scale. But you will also notice that the right index of the first B scale (the middle index of the double B scale) coincides with 4 of the second A scale, so that by sliding the right index of the first scale under the first factor on the second scale A



and reading backward on the first scale B you get the same result. We have thus used really one scale on the

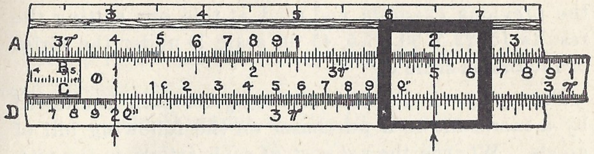



FIG. 11.

slide as well as on the stock, namely, the first scale on the former and the second scale on the latter. We can, however, just as well substitute the first A scale for the second by placing the middle index of B under 4 of (first) A scale and read backward, 20 on (first) A scale over 5 on the (first) B scale. Thus it appears that we do not really need the second scale, but can perform the operation as well on the C and D (single) scales by using the right hand index and reading backward. It is, therefore, obvious that, when the sum of two factors is shorter than the rule, the left index of C is used, and if longer, the right one.

**Example.**— Sought the product of 7.5×4. As the sum of the two distances representing the two factors exceeds the length of the rule, the right index of the scale C is brought over 7.5 on D and underneath 4 on scale C, the product 30 is found (Fig. 12) on scale D.

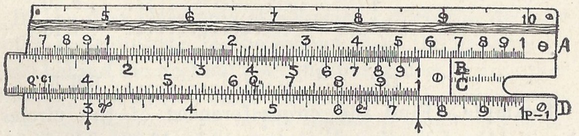



FIG. 12.

**Summary.**—To find the product of two factors, set the right or left index of scale C over the first factor on scale D, then the product will be found on scale D underneath the second factor on scale C.

If one wishes to multiply a series of factors by another common factor, it is obviously necessary to set the rule but once, thus continuing the last example, by multiplying successively the numbers 2, 3, 4, 5, 6, for instance, with the common factor 7.5, we set the rule as in the previous example and read at this one setting the results 15, 22.5, 30, 37.5, 45 on scale D under the series of factors on scale C.

**Example.**—A discount of 12½% for cash is allowed by a merchant to a purchaser on the following bill of goods. What is the net cost of each article to the purchaser and the net amount of the whole bill?

| | |
|---|---|
| 1 Set Drawing Instruments............ | $32.00 |
| 1 Drawing Table........................ | 18.00 |
| 6 T Squares @ 1.50 .................... | 9.00 |
| 2 Steel Tapes @ 11.50.................. | 23.00 |
| 1 Proportional Divider................. | 21 00 |
| 2 Architects' Scales @ 2.00............. | 4.00 |
| 1 Set Celluloid Triangles............... | 2.00 |
| 1 Beam Compass, in case............... | 13.00 |
| Total.................... | $122.00 |

**Solution.**—The net amounts are obtained by multiplying with .875. Setting the right index of C over .875 on D, we read:

| | | | |
|---|---|---|---|
| On D, under | 32 | (on C)................ | $28.00 |
| " " | 18 | " ................ | 15.75 |
| " " | 9 | " ................ | 7.875 |
| " " | 23 | " ................ | 20.125 |
| " " | 21 | " ................ | 18.375 |
| " " | 4 | " ................ | 3.50 |
| " " | 2 | " ................ | 1.75 |
| " " | 13 | " ................ | 11.375 |
| " " | 122 | " ................ | 106.75 |

**Example.**—Find the circumferences of the circles having the following diameters, 12.5′, 16.25′, 23′, 31.3′, 55′, 78′, 81′, 92.5′.

**Solution.**—The circumference is obtained by multiplying the diameter with $\pi=3.1416$. For convenience, the space representing this figure is marked $\pi$ on the slide rule in various places.

Set the left index of C over $\pi$ on D and read successively:



| | | | |
|---|---|---|---|
| On D. under | 12.5 | (on C)..... | 39.3′ |
| " " | 16.25 | " ..... | 51.00′ |
| " " | 23.00 | " ..... | 72.25′ |
| " " | 31.3 | " ..... | 98.3′ |

Set the right index of C over $\pi$ on D and continue to read successively:

| | | | |
|---|---|---|---|
| On D, under | 55 | (on C)..... | 173′ |
| " " | 78 | " ..... | 245′ |
| " " | 81 | " ... | 254.5′ |
| " " | 92.5 | " ... | 290.5′ |

It is obvious that the upper scales A and B may be used in the same manner as the lower scales C and D, and in the case of the multiplying of a series of numbers by the same factor, it may prove more convenient to use them in preference, at the cost of accuracy. Thus in the last example, all the results may be read off at a single setting with the upper scales instead of two settings that were necessary with the lower scales. A trial will readily show this.

**Position of the Decimal Point.**—In many cases the decimal point can be determined by a mere glance at the problem. Doubts will, however, arise in more complex calculations, and the following will give the necessary information for arriving at exact results. There are two kinds of decimals:

1. Such with figures before the decimal point.
2. Such without figures before the decimal point.

In the first class only the figures before the decimal point are counted; in the second class the cyphers (if any) behind the decimal point up to the first figure other than 0. The figures counted in the first class are called plus (+) places, those in the second minus (—) places. Thus in the decimal 5430.004, for instance, there are four + places, while in the decimal .004 there are two — places. The above agreed upon, the following two rules are used to determine the location of the decimal point.

1. If the product is read off on the **left of the index,** that is, when setting has been made with the right index of C, the number of places in the product equals the **sum of the places of the factor.**

2. If the product is read off on the **right of the index,** that is, when the setting has been made with the left index of C, the number of places in the product is equal to the **sum of the places of the factors minus 1.**

The symbol P—1, at the right end of the scale D, serves as a reminder of these two rules to the operator. Although these rules refer especially to the scales of C and D, they are applicable to the scales of A and B as well, when one remembers that instead of using the right index in setting for a multiplication one may always use the left of the first scale and read the result in the second. The rules may thus be stated for these scales as follows:

(1a). If the product is read off to the **right** of the middle index of scale A, that is, on the second scale, the number of places in the product equals the **sum of the places of the factors.**

(2a). If the product is read off on the **left** of the middle index of scale A, that is, on the first scale, the number of places in the product is equal to the **sum of the places of the factors minus 1.**

Conversely, as the use of the right index in scales B and C is really but the use of an imaginary second scale, the four rules may be represented in the following diagram:

| PRODUCT READ ON | PLACES IN PRODUCT |
|---|---|
| First scale | Sum of places of factor —1 |
| Second scale | Sum of places of factors |

**Examples:**

| FACTORS | PRODUCT | READ | PLACES OF PRODUCT |
|---|---|---|---|
| 5.43×1.47= | 7.98 | First | 1+1—1=+1 |
| .27×57.6= | 15.55 | Second | 0+2=+2 |
| .42×.161= | .0676 | First | 0+0—1=—1 |
| .058×37.6= | 2.18 | Second | —1+2=+1 |
| 5430×.00013= | .706 | First | 4+(—3)—1=0 |
| .00062×.0000054= | .00000000335 | Second | —3+(—5)=—8 |



## VI.—DIVISION.

**Principle.**—If the logarithm of a number is **deducted** from that of another number, the difference corresponds to the **quotient** of the two numbers.

**Mode of Operation.**—Division, like multiplication, and for the same reason, is preferably performed on the lower scales C and D. As apparent from the preceding explanations, the quotient of two numbers is obtained by deducting the graphical distance representing the divisor from that of the dividend. The difference found is the quotient. Test this by subtracting the space marked 3 on scale C from the space marked 6 on scale D by sliding the 3 on C over the 6 on D, then read off the quotient 6 ÷ 3=2 on D under the left index of C.

**Example.**—Sought the quotient of 2.83 ÷ 2.

Set 2 on scale C over 2.83 on scale D and below left index of C find the difference, i. e., the quotient 1.415 on scale D. (Fig. 13),

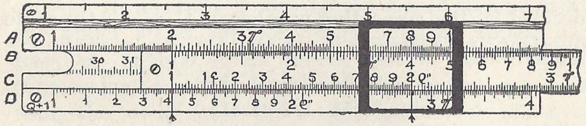


Fig. 13.

In performing divisions it will be found very convenient to use one of the hair-lines of the indicator to locate the dividend and to bring the division in coincidence with it.

In the two examples thus far given, the divisor was smaller than the dividend. If the reverse is the case, the left index of the scale C will be beyond the scale D, but it will be found that the right index of C is over the proper result. Thus, if the quotient of 72 ÷ 9 is sought, the result 8 will be found on D under the right index of C. Fig. 14 demonstrates this operation.

**Summary.**—To find the quotient of two numbers, the divisor on C is brought over the dividend on scale D and under the left or right index of C, the quotient will be found on D.

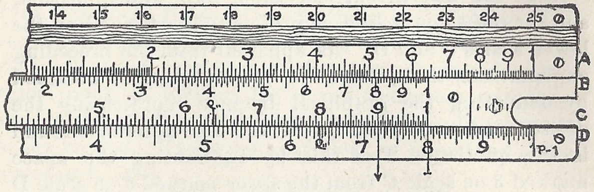


Fig. 14.

Returning to the first simple example 6÷3=2, it is evident that the quotient 2 may be the result of an infinite number of divisions, such as, 4÷2=, 6÷3=, 8÷4=, 10÷5=, etc., and since, if the rule is truthful, all these pairs of numbers must coincide, we discover what is one of the most important features of the slide rule, namely, that all the coinciding numbers of two adjoining scales bear the same relation, so that every number on D, divided by the coinciding number on C, yields the same quotient; for instance, in the above examples we find:

$$2.264 \div 1.6 = 1.415$$
$$3.254 \div 2.3 = 1.415$$

This leads to a convenient way of dividing a series of dividends by the same divisor. For example, to find the 3d part of the numbers 36, 45, 6, 90, 108, 213, we set the rule as if we were to divide 1 by 3, that is, we bring into coincidence 3 on C with 1 (left) on D and read off

On D, under 36 (on C)......12
" " 45 " ......15
" " 6 " ...... 2
" " 90 " ......30

Next we bring into coincidence 3 on C with 1 (right) on D and read further

On D, under 108 (on C)......36
" " 213 " ......71



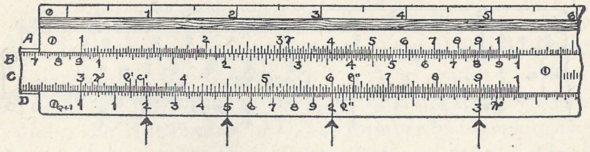


Fig. 15.

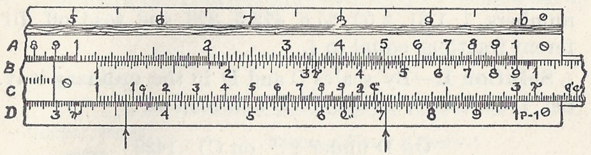


Fig. 16.

It is obvious that the upper scales, A and B, may be used instead of the lower ones, C and D, and in setting for a table of divisions it may be found more convenient, of course at the cost of accuracy, to use them, as all the results may be read off at a single setting; the second halves of the double scales avoid the necessity of using the right index and two settings, used with the lower scales. Incidentally it may be mentioned that in using the upper scales the coinciding numbers appear like fractions having equal values, that is, the dividend (numerator) over the divisor (denominator), while in using the lower scales the reverse is the case, that is to say, using scales A and B, the following fractions of equal value may be read off (corresponding to the last example).

$$\frac{1}{3}, \frac{12}{36}, \frac{15}{45}, \frac{2}{6}, \frac{3}{9}, \frac{36}{108}, \frac{71}{213}, \text{etc.}$$

while in using scales C and D, these fractions would appear reversed, thus (Figs. 15 and 16),

$$\frac{3}{1}, \frac{36}{12}, \frac{45}{15}, \frac{6}{2}, \frac{9}{3}, \frac{108}{36}, \frac{213}{71}, \text{etc.}$$

This would seem at first sight a slight drawback for scales C and D, but it is possible to make the numbers appear like regular fractions in these scales also, by considering the slide stationary and the rule moving, and, in other words, by setting the dividend (numerator) on the slide scale C over the divisor (denominator) on the rule, scale D.

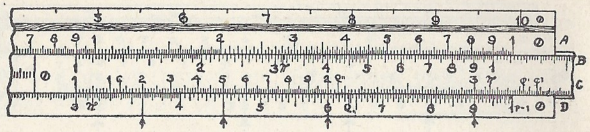


FIG. 17.

**Examples.**—Find the seventh part of the following numbers, 1, 1.31, 2.02, 35.6, 472.5, 823, and read off different fractions equal to $\frac{1}{7}$.

**Solution 1.**—By scales C and D in the ordinary way. Set 7 on C over 1 (left) of D and read off:

On D under 1 (on C) .1429
" " 823 " 117.6

Set 7 on C over 1 (right) of D and continue to read:

On D under 1.31 (on C) .187
" " 2.02 " .2886
" " 35.6 " 5.09
" " 472.5 " 67.5

At the same setting one may read off the following fractions equal to $\frac{1}{7}$:

$\frac{C}{D}$ $\left(\frac{105}{15}\right)$ $\left(\frac{112}{16}\right)$ $\left(\frac{119}{17}\right)$ $\left(\frac{14}{2}\right)$ $\left(\frac{147}{21}\right)$ $\left(\frac{175}{25}\right)$ $\left(\frac{28}{4}\right)$ $\left(\frac{525}{75}\right)$ $\left(\frac{665}{95}\right)$

$\frac{D}{C}$ $\left(\frac{15}{105}\right)$ $\left(\frac{16}{112}\right)$ $\left(\frac{17}{119}\right)$ $\left(\frac{2}{14}\right)$ $\left(\frac{21}{147}\right)$ $\left(\frac{25}{175}\right)$ $\left(\frac{4}{28}\right)$ $\left(\frac{75}{525}\right)$ $\left(\frac{95}{665}\right)$

**Solution 2.**—By scales C and D. Set 1 of C over 7 on D and read results on C in two (right and left) settings.

**Solution 3.**—By scales A and B. Set 7 of B under (left) of A and read results on A in one setting.

**Position of Decimal Point.**—The position of the decimal is ascertained by the index, by means of which the quotient is read off according to the following rules:

Rules for Scales C and D:

1. If the result lies on the left of the setting, that is,



under the left index of C, the number of places of the quotient is equal to the number of places of the dividend minus that of the divisor plus 1.

2. If the result lies to the right of the setting, that is, under the right index of C, the number of places of the quoticnt is equal to the number of places of the dividend minus that of the divisor.

The mark Q+1, at left end of scale D, serves as a handy reminder to the operator.

**Summary:**

| Quotient read off under | Number of Places of Quotient |
|---|---|
| Left index | Places of dividend — Places of divisor+1 |
| Right index | Places of dividend — Places of divisor |

**Examples:**

| Problem | Quotient | Places of Quotient | Reading taken under Index |
|---|---|---|---|
| 7.63÷1.45 | 5.26 | 1—1+1=+1 | Left |
| 2.85÷9.02 | 0.316 | 1—1=0 | Right |
| 584.÷33.7 | 17.34 | 3—2+1=+2 | Left |
| 295.÷3420 | 0.0863 | 3—4=—1 | Right |
| 0.47÷57.6 | 0.00816 | 0—2=—2 | Right |
| 0.032÷121 | 0.000264 | —1—3+1=—3 | Left |
| 5.31÷.0422 | 12.58 | 1—0+1=+2 | Left |
| 79.2÷0.033 | 2400. | 2—(—1)+1=+4 | Left |
| 0.846÷0.411 | 2.06 | 0—0+1=+1 | Left |
| 0.027÷0.081 | 0.333 | —1—(—1)=0 | Right |
| 0.0091÷0.000032 | 284.5 | —2—(—4)+1=+3 | Left |
| 0.000163÷0.0702 | 0.00232 | —3—(—1)=—2 | Right |

Rules for Scales A and B:

The same rules apply to scales A and B as long as the quotient is read in the same half scale as the dividend. This is mentioned solely because it will be found convenient in compound calculations (of which we shall speak later) to read the quotient in the other half scale.

## VII.—PROPORTIONS.

For the solving of proportions, use scales A and B. For operations with the slide rule, arrange the proportions in the form of an equation of two fractions (ratios), both the numerator and the denominator of the first fraction, and either the numerator or the denominator of the second being known, the unknown fourth term being sought as in

$$(1)\ \frac{a}{b} = \frac{c}{x} \text{ or } (2)\ \frac{a}{b} = \frac{x}{c}$$

in which a, b and c are the three known terms and x the unknown. It is always possible to bring proportions into one or the other of the two forms; care is to be taken only to place the unknown quantity in the right place. This is very easy when one considers that when a is greater than b, the numerator of the second fraction must always be greater than the denominator and *vice versa*. Take for instance the problem:

If 9 yards of silk cost $27, what will 18 yards cost?

The left ratio will be $\frac{9\text{ (yds)}}{18\text{ (yds)}}$ and since the greater number of yards will command a greater amount of money, and this greater amount is sought, the unknown quantity x is to be placed in the position corresponding to the greater number of yards, that is, in the denominator of the second fraction thus: $\frac{9\text{ yds}}{18\text{ yds}} = \frac{27(\$)}{x(\$)}$.

Such proportions are called direct proportions. Take, on the other hand, the following problem: If 6 men can dig a ditch in 48 days, in what time can 8 men dig it?

Eight men can do the work in less time than 48 days, consequently the answer is less than 48 days and the proportion should be written $\frac{6\text{ men}}{8\text{ men}} = \frac{x\text{ days}}{48\text{ days}}$, that is, the result or unknown term must be placed in the second ratio corresponding to the smaller quantity in the first. Such proportions are called inverted proportions.



The solving of such problems on the slide rule is exceedingly simple. It resolves itself into finding to a given fraction another fraction equal in value whose numerator or denominator is given. It is, therefore, only necessary, in the light of what has been said in section VI., to follow the following rule:

**Rule.**—Bring the numerator of the first, or given fraction, on the scale A in coincidence with the denominator on the scale B and read the result on scale B under the given third term on scale A in case 1. Read on scale A over given third term on scale B in case 2.

Expressed symbolically, set and read:

$$(1)\ \frac{A}{B} \Big\| \frac{a}{b}\ \frac{c}{x}$$

$$(2)\ \frac{A}{B} \Big\| \frac{a}{b}\ \frac{x}{c}$$

Diagrams Figs. 18 and 19 show the settings of the rule for the two examples given, the results being 54 and 36 respectively.

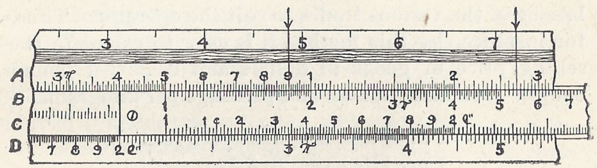



FIG. 18.

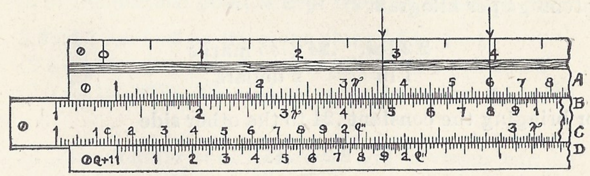



FIG. 19.

**Example.**—When 7 meters are equal (nearly) to 23 feet, how many feet are 15, 18, 20.5, 3.56 meters?

**Solution.**—Set 23 on B under 7 on A and read:

| | | | |
|---|---|---|---|
| On B under | 15 | (on A) | 49.4′ |
| " " | 18 | " | 59.2′ |
| " " | 20.5 | " | 67.5′ |
| " " | 3.56 | " | 11.7′ |

Here it is well to emphasize that the finding of the unknown term of a proportion is the calculation oftenest performed on the slide rule, and a vast number of problems can be solved by it. The following are two more examples:

The ratio between the diameter of a circle and its circumference, $\pi$, may be approximately given by the common fraction $\frac{355}{113}$. (This fraction is very easily remembered by the following rule: Write the first three odd figures, 1, 3, 5, down twice, thus: 113355, and use the first three of these six figures as the denominator and the last three as the numerator). To find the circumference of any circle, place 113 on B under 355 on A and read circumferences on A over diameter on B.

Such ratios as the one just given are called gauge points, and may be established for all sorts of calculations in the various trades to suit the operator. Thus, for instance, by this method it is easy to calculate the selling price of goods in dollars and cents per pound bought at so many marks and pfennigs per kilogram.

1 kilogram is equal to 2.2 lbs.
1 mark is equal to $.24

Therefore we have, if the German price is, say, 45 pfennigs per kilogram

$$\frac{2.2 \text{ lbs.}}{1 \text{ lb}}=\frac{.24\times.45 \text{ marks}}{x \text{ dollars}}$$

or bringing the constant .24 to the other side

$$\frac{2.2}{.24}=\frac{.45}{x}$$

Set 24 on B under 22 on A and read dollars, x on B under marks on A in this particular example $.0491.

For the finding of the unknown term of a proportion,



we have recommended scales A and B, solely because these scales always permit the result to be obtained in one setting of the rule in the manner above explained. This is not the case when the scales C and D are used. Thus, for example, for instance, illustrated in Fig. 18, one must perform the two operations, multiplication and division, which in reality compose the finding of the fourth term separately, according to the formula

$$x=\frac{27\times18}{9}$$

necessitating two settings, as a trial will convince.

As to the decimal point in the sought fourth term of a proportion, the reader is referred to the following chapter.

## VIII.—CONTINUED MULTIPLICATION AND COMBINED MULTIPLICATION AND DIVISION.

When the product of more than two fractions is to be found, the multiplications are performed one after another, each product being fixed by the indicator, but not read off, except the final result. Thus to multiply the factors 2.3, 4.1, .06, .003, place the left index of C over 2.3 on D, slide the indicator over 4.1 on C, slide right index of C under the indicator, slide indicator over 6 on C, slide right index of C under the indicator and read final result, .001697 on D under 3 of C.

While the various operations are performed, the position of the decimal point is kept track of mentally, thus:

$2.3\times4.1$ read off on the right gives $1+1-1=1$
$(2.3\times4.1)\times.06$ read off on the left gives $1+(-1)=0$
$(2.3\times4.1\times.06)\times.003$ read off on the left gives $0+(-2)=-2$.

In like manner multiplication and division can be performed one after another by means of the indicator without noting intermediate results. Thus, for instance, to solve the following problem:

$$x=\frac{27.02\times1.82}{1.3\times9.81}$$

Perform the multiplication:

$27.02 \times 1.82$ and note that the result held by the indicator will have $2+1-1=2$ places. Next perform the division:

$(27.02 \times 1.82) \div 1.3$ and note for the decimal point $2-1+1=2$.

Last perform the division:

$\left(\dfrac{27.02 \times 1.82}{1.3}\right) \div 9.81$ and note for the decimal point $2-1=1$ and the final result 3.86.

When finding the fourth term of a proportion the decimal point is now easily located, inasmuch as the operation is a combined multiplication and division. It is only necessary to imagine the proportions written in the forms:

$$(1)\ x = \frac{b}{a} \times C \qquad (2)\ x = \frac{a}{b} \times C$$

Thus in the example given above in setting the rule for

$$\frac{A}{B} \Bigg\| \frac{2.2}{.24} = \frac{.45}{x}$$

we actually perform the operations

$$\frac{A}{B} \Bigg| \frac{.24}{2.2} \text{ and } \overset{B}{\frac{.24}{2.2}} \times \overset{A}{.45}$$

in which the first yields for the decimal point $0-1+1=0$ and the second $0+0-1=-1$, so that the result is .0491 dollars.

## IX.—SQUARES.

**Principle.**—If the logarithm of a number is multiplied by another number, the result obtained is (not the logarithm of the product but) the logarithm of the power of the first number with the second number as exponent. Thus, doubling a logarithm of a number is multiplying it by itself, which is performed on the slide



rule by adding equal spaces of the logarithmic scales. Any square can, therefore, be found on either of the scale-pairs A, B, and C, D by placing the index of C in coincidence with the number on D and reading under the same number on C its square on D.

**Mode of Operation.**—For greater convenience, however, the spaces of the lower scales, C and D, are twice the length of the upper ones, A and B, from which it is apparent that the numbers of the top scale A represent the squares of the coinciding numbers on the bottom scale D, and *vice versa;* every number on the bottom scale D the square root of the number on the top scale A with which it is in alignment. The readings are generally taken with the indicator, but in cases where the altering of the position of the latter would impede the operation, as in compound calculations, they can also be read by the right or left index of the slide.

**Position of the Decimal Point.**—1. If the square of a number, fractional number or fraction, appears on the upper left scale A, the number of places of the square is twice that of the first power minus 1.

2. If the square falls on the right upper scale A, the number of its places is twice the number of its first power.

**Examples:**

| Problem | Places of First and Second Power | | Side of Scale Squares Found on | Square |
|---|---|---|---|---|
| $1.92^2$ | +1 | $2\times1-1=+1$ | Left | 3.694 |
| $30.5^2$ | +2 | $2\times2-1=+3$ | " | 932. |
| $2090^2$. | 4 | $2\times4-1=+7$ | " | 4368000. |
| $.164^2$ | +0 | $2\times0-1=-1$ | " | 0.0269 |
| $.00291^2$ | −2 | $2\times(-2)-1=-5$ | " | .00000847 |
| $4.1^2$ | +1 | $2\times1=+2$ | Right | 16.8 |
| $557^2$. | +3 | $2\times3=+6$ | " | 310000 |
| $.912^2$ | +0 | $2\times0=+0$ | " | 0.832 |
| $.000931^2$ | −3 | $2\times(-3)=-6$ | " | .000000535 |

## X.—SQUARE ROOTS.

**Principle.**—If the logarithm of a number is divided by another number, the result is (not the logarithm of the quotient but) the logarithm of the root of the first number with the second number as radical index.

**Mode of Operation.**—It is clear at once, that if the number on the upper scales A and B are the squares of the coinciding numbers on C and D, the latter are conversely the square roots of the former.

**Rule.**—To find the square root of a given number, fractional number or fraction, the figures preceding the decimal point are divided into groups of two places each, starting at the decimal point and proceeding to the left. Fractions are divided up the same way, in groups of two places each, behind the decimal point, starting at the latter and proceeding to the right. Radicands, the square root of which is sought, are then read on left scale A, if the last group on the left consists of one place only. If this group consists of two places read off on right scale A.

The same rule also applies to fractions, only that in this case the first group on the right of the decimal point containing a figure other than cyphers has to be consulted. If the same contains a cypher and a figure, the number has to be read on left scale A, and if it consists of two figures, it has to be read on the right scale A. The number of places of the square root of a fractional number is equal to the number of groups of figures on the left of the decimal point. The square root of a fraction is again a fraction with as many cyphers after the decimal point as there are groups of cyphers in the radicand.

In the graphical exemplification of these rules, on the next page, the radicand is shown divided in groups of two figures each by commas and the ultimate group printed in heavier type to make it more readily distinguishable.



**Examples:**

| Problem | Setting on Scale | Number of Places in Ultimate | in Root | Result |
|---|---|---|---|---|
| $\sqrt{\mathbf{1}'.74}$ | Left A | 1 | +1 | 1.32 |
| $\sqrt{\mathbf{15}'.2}$ | Right " | 2 | +1 | 3.9 |
| $\sqrt{\mathbf{15}'20'.}$ | " " | 2 | +2 | 39.0 |
| $\sqrt{\mathbf{9}'27.}$ | Left " | 1 | +2 | 30.45 |
| $\sqrt{\mathbf{7}'45'00'}$ | " " | 1 | +3 | 273. |
| $\sqrt{.\mathbf{08}'53'}$ | " " | 1 | +0 | .292 |
| $\sqrt{.\mathbf{85}'1}$ | Right " | 2 | +0 | .922 |
| $\sqrt{.00'\mathbf{48}'}$ | " " | 2 | +0 | .0694 |
| $\sqrt{.00'00'\mathbf{09}'92}$ | Left " | 1 | −2 | .00315 |

## XI.—CUBES.

**Principle.**—From the principle stated in Chapter IX., it is evident that the cube of a number is obtained on the slide rule by adding together three equal spaces of the logarithmic scale.

**Mode of Operation.**—The work is, however, shortened by the arrangement of the scales mentioned. The upper scales, A, B, give the square of the numbers of scales C, D. Therefore, in order to obtain the cubes, it is simply necessary to add to the squares (double space) another single space. This will be best shown in an example.

**Example.**—Required the cube of 2; set left index of C over 2 on scale D, and in alignment with 2 on scale B the cube 8 will be found on left scale A, as shown in Fig. 20. The fundamental principle of this operation is very plain. By adjusting the left index of scale C to 2 on scale D the square of 2=4 is obtained on scale A over the left index of B, and this, multiplied by 2 on the upper scale B, yields on A $2 \times 2 \times 2 = 8$ = the cube of 2.

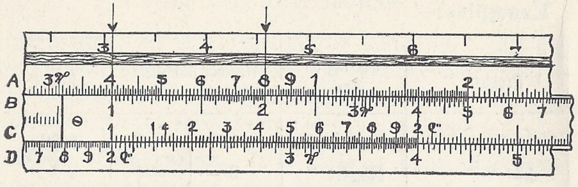



FIG. 20.

**Position of the Decimal Point.**—1. If the left index mark of C is employed in making the first setting and the cube is read on the upper left scale A the number of places in the result is three times that of the first power minus 2.

2. If the left index mark of C is again employed for the initial setting and the power appears on the right scale A, the number of figures of the cube is three times that of the first power minus 1.

3. If the right index of C is used and the cube read on the upper left scale A, the number of figures in the result is three times those composing the first power.

**Notice.**—For all these operations, only the upper left scale B of slide is employed.

The same operation can also be performed with the slide inverted, i. e., the slide is pulled out, reversed end for end and reinserted with the same face up, scale B adjacent to D and scale C to A. The first power on scale D is brought into coincidence with the same number on scale B, left inverted, whereupon the cube can be read off under one of the indexes on either scale A left or right.



**Examples:**

| Problem | First Power | Setting by Index of C | Reading Taken on Upper Scale | Places in Cube | Cube |
|---|---|---|---|---|---|
| $1.32^3$ | +1 | Left | Left A | 3×1—2=+1 | 2.30 |
| $32.5^3$ | +2 | " | Right A | 3×2—1=+5 | 34300. |
| $567^3$ | +3 | Right | Left A | 3×3=+9 | 182000000 |
| $.624^3$ | +0 | " | " | 3×0=0 | .243 |
| $.421^3$ | +0 | Left | Right A | 3×0—1=—1 | .0746 |
| $.175^3$ | +0 | " | Left A | 3×0—2=—2 | .00536 |
| $.0206^3$ | —1 | " | " | 3×(—1)—2=—5 | .00000874 |
| $.0000432^3$ | —4 | " | Right A | 3×(—4)—1=—13 | .0000000000008 |
| $.000957^3$ | —3 | Right | Left A | 3×(—3)=—9 | .000000000876 |

## XII.—CUBE ROOTS.

**Principle.**—From the principle stated in Chapter X, it is apparent that the logarithm of a cube root of a number is formed by dividing the logarithm of a number into three equal parts.

**Mode of Operation.**—To find the cube root of a given number with the slide in its normal position, the indicator is set to the radicand on upper scale A (see directions below), the slide is then shifted, either right or left, until the same number appears simultaneously on left scale B in coincidence with the hair-line on the indicator and on scale D in alignment with one of the indexes of scale C. The position of the decimal point of the root sought is determined by dividing the figures before, or the cyphers behind the decimal point in the radicand in groups of three places each. Each one of these groups represents one figure or cypher respectively in the root, as explained under "Square Roots."

The following rule will assist materially in obtaining precise readings:

### (a). With the Slide in its Normal Position.

1. All operations performed by co-working the upper scales A with the left scale of B only.
2. If the ultimate group of figures or cyphers consists of three places, the indicator has to be aligned with the radicand on upper left scale A and the result read off by means of the right index of C.
3. If the ultimate group is composed of two figures, the indicator is set to the radicand on upper right scale A and the reading taken under the left index of C.
4. If the ultimate group consists of only one figure, the indicator is adjusted to the radicand on the upper left scale A and the reading taken under the left index of C.



**Examples:**

| Problem | Ultimate Group | Setting on Upper Scale A | Result Under Index of Scale C | Places in Cube Root | Cube Root |
|---|---|---|---|---|---|
| $\sqrt[3]{1'43}$ | 1 Place | Left | Left | +1 | 1.126 |
| $\sqrt[3]{432'}$ | 3 " | " | Right | +1 | 7.56 |
| $\sqrt[3]{99.'5}$ | 2 " | Right | Left | +1 | 4.63 |
| $\sqrt[3]{4.'530}$ | 1 " | Left | " | +2 | 16.55 |
| $\sqrt[3]{2.'460'000}$ | 1 " | " | " | +3 | 135.0 |
| $\sqrt[3]{57'600}$ | 2 " | Right | " | +2 | 38.6 |
| $\sqrt[3]{0.'357}$ | 3 " | Left | Right | +0 | 0.710 |
| $\sqrt[3]{0.046'2}$ | 2 " | Right | Left | +0 | 0.358 |
| $\sqrt[3]{0.'001'61}$ | 1 " | Left | " | +0 | 0.1172 |
| $\sqrt[3]{0.'000'846}$ | 3 " | " | Right | —1 | 0.0946 |
| $\sqrt[3]{0.000'000317'}$ | 3 " | " | " | —2 | 0.00516 |

### (b). With the Slide Inverted.

When a series of cube roots are sought, the process of finding them is simplified by using the slide inverted (as shown by examples at the end of Chapter XI.)

The following essential rules have to be strictly observed:

1. If the ultimate group consists of three figures, the right inverted index of C is set to the radicand on left upper scale A.

2. If the ultimate group is composed of two figures, the left inverted index of C is set to the radicand on the upper right scale A.

3. If the ultimate group consists of one figure only, the left inverted index of C is set to the radicand on the upper left scale A.

4. By inspection of the adjacent bottom scales B inverted and D, two equal numbers will be found in coincidence, the cube root sought.

The position of the decimal point is ascertained by the number of groups of three places each composing the radicand as with the slide in its normal position (see preceding instructions). The examples given in the preceding chapter also apply to calculations with the slide inverted.

**Powers higher than cubes** can, of course, be found by repeated multiplication, thus the fourth power of a number is found by multiplying its square with its square, the fifth by multiplying its cube with its square. Of **roots higher than cube roots,** some can be found directly, others not; thus the fourth root is found by extracting the square root twice, the sixth by extracting consecutively the cube and the square roots, and so with all roots the indices of which are multiples of 2 and 3. For finding other roots, another method is used with the help of the scale L of logarithms, that appears in the middle of the bottom of the slide. This same method may be used for all powers and roots higher than cubes and cube roots respectively. Before giving this method, however, it will be necessary to explain the nature of scale L.

## XIII.—LOGARITHMS.

From what has been said in Chapter IV, the reader knows that the spaces on the slide rule scales, from the left index to consecutive numbers, are the logarithms of these numbers laid off to a certain scale of equal parts, the unit of which is the length from index to index. To find the logarithm of any number, therefore, it is necessary only to measure by that certain scale of equal parts the spaces on the slide rule. Such a measuring scale of equal parts is provided in the scale L above mentioned,



in the middle of the reverse side of the slide. It is divided into 10ths, 100ths and 500ths, so that 1000th must be estimated by halving the smallest divisions on this scale. It is intended to be used normally with the slide in its regular position, that is, the scale L out of sight, the logarithms on it being read by an index mark on the right hand notch on the back of the rule, as follows: The initial index of scale C is brought into alignment with the number on scale D, whose logarithm is wanted. The rule turned over and the mantissa of the corresponding logarithm read on scale L above the index mark on the lower edge of the notch. A logarithm is always a decimal, the fractional part (behind the decimal point) being called the **mantissa,** and since any number on the scale D may represent any power of 10 multiplied with it, the mantissa for any number is the same, whether representing units, tens, hundreds, etc., thus the mantissa of log. 3 is .477, no matter whether the number is .003, .03, 3, 30, 300, 3,000, etc. Which of these numbers is meant in each case is told by the integer (before the decimal point), called the **characteristic** of the logarithm, and this is always **one less** than the number of positive places of the number and is positive, thus:

Log. 3 = .477
Log. 30 = 1.477
Log. 300 = 2.477

and is always **one more** than the number of negative places, or cyphers, behind the decimal point and is **negative,** thus:

Log. .3 $= \bar{1}.477$
Log. .03 $= \bar{2}.477$
Log. .003 $= \bar{3}.477$

It will be noticed that in the case of a negative characteristic, the minus sign is written **over the characteristic** and not before it, to indicate that the characteristic alone is negative and not the whole expression.

It is evident that the foregoing enables one to read on scale D any number corresponding to a given logarithm.

**Examples.**—Sought the logarithm of a given number.

| Given Number | Characteristic | Mantissa Read on Scale L | Log. Sought |
|---|---|---|---|
| 243. | +2 | +0.386 | 2.386 |
| 1.76 | +0 | +0.2455 | 0.2455 |
| 0.032 | −2 | + .505 | $\bar{2}.505$ |

Sought the number corresponding to a given logarithm.

| Given Log. | Mantissa Set on Scale L | Characteristic | Places in Number | Number Sought |
|---|---|---|---|---|
| 4.321 | 321 | +4 | +5 | 20950 |
| $\bar{2}.863$ | 863 | −2 | −1 | 0.0729 |
| 1.952 | 952 | +1 | +2 | 89.5 |

When a continuous series of logarithms has to be read off, the operation can be accelerated by pulling out the slide and reinserting it reversed, end for end, with its lower face bearing the scale upwards, i. e., scale T adjacent to scale A and scale S adjoining scale D. After the indexes are brought into alignment, consecutive series of logarithms can be read off by means of the indicator and, *vice versa*, natural numbers corresponding to the logarithms on scale L.

## XIV.—POWERS AND ROOTS.

Powers and roots other than squares, cubes, square roots and cube roots are determined on the slide rule by means of the scale L of logarithms on the back of the slide. This is accomplished by reading off the logarithm of the number that is to be raised, or of the radicand, the root of which is to be taken, and multiplying or dividing it by the exponent. The product or quotient obtained is the logarithm of the power or root sought, the **mantissa** of which is then set off on scale L, whereupon the power or root are read off on scale D, underneath the left index on scale C.

The position of the decimal point is obtained as explained in the foregoing chapter (Logarithms).



**Example.**—Find $6^5$.

**Solution.**—Set the left index of C above 6 on scale D, turn over the rule and read off logarithm .778, set right index of C over .778 on scale D and underneath 5 on scale C find the product, i. e., the logarithm 3.89; to convert this into the corresponding number, turn over the rule again and set the mantissa .89 on scale L to the under mark on notch and take the final reading 7776, the power sought, on scale D under the index of C.

The position of the decimal point is ascertained from the characteristic +3 of the logarithm 3.89, which yields four places in the number sought.

**Example.**—Find $\sqrt[7]{3652}$.

**Solution.**—Set left index of C over 3652 on D and read logarithm 3.5625 on L. Set 7 on C over 3.5625 on D and read .509 on D. Set mantissa 509 on L to index mark on notch and turning over read 3.23, the root on D under the left index of C.

## XV.—TRIGONOMETRICAL FUNCTIONS.

On the under side of the slide are two additional scales besides the one of equal parts (logarithm) L, namely, scales "S" of sines from 34′ to 90° and a scale T of tangents of angles from 5°43′ to 45°. These scales are regular logarithmic scales, such as scales A, B, C and D, the spaces up to the various figures denoting angles being the logarithms of the sines or tangents respectively of these angles, measured by a scale of a certain unit. The scale S of sines is laid off with the same unit as scale A, while the scale T of tangents corresponds to scale D, and they are intended to be used in conjunction with these scales, in the same way as the regular slide scales B and C are used with the rule scales A and D.

**Sines.**—Having placed the scale S in alignment with scale A, one can read on A the sine of any angle on S, thus, over 30° on S, for instance, find .5 on A and *vice versa*. The numerical values of the sines read on the left half scale of A (from sine 34′ to sine 5° 44′) lie between .01 and .1, and those read on the right half of A (from sine 5° 44′ to sine 90°) between .1 and 1.

**Examples :**

(1) Find the sine of 30° 25′.

Read over 3° 25′ on S, 596 on A, and since this number is on the first half of A, the sine of 3° 25′ is .0596.

(2) Find the sine of 16° 50″.

Over 16° 50′ on S read 29 on A, and since this is found on the second half of A, the sine sought is .29.

(3) Find the angle the sine of which is .0435.

Since the given sine is less than 1, the angle is found on S under 435 of the **first** half scale **A,** and is 2° 30′.

(4) Find the angle the sign of which is .275..

Since the sine is greater than .1, the angle is found on S under 275 of the **second** half scale **A,** and is 15° 58′.

**Cosines.**—The cosine of an angle is equal to the sine of its complement as expressed in the formula :

cos A=sine (90°—A), so that a cosine may be easily found by mentally subtracting the angle given from 90° and finding the sine of the difference according to the rules for sines.

**Examples :**

(1) Find the cosine of 43° 30′.

cos 43° 30′=sine (90°—43° 30′)=sine 46° 30′=.725.

(2) Find the angle the cosine of which is .231.

Treat the given number as a sine ; look for the angle under 231 on second half of scale A and find 13° 20′. Subtract this from 90° and have for the angle sought 76° 40′,

**Tangents and Cotangents.**—In the same manner as sines and cosines are read on A over angles on S, and *vice versa*, so tangents and cotangents are read on scale D under angles on T. Scale T, however, contains only the angles 5° 43′ to 45°, corresponding to the tangents .1 to 1 on scale D. The following set of formulas serve to find the tangents and cotangents of any angle from 5° 43′ to 84° 17′.



| | | | |
|---|---|---|---|
| A | For angles A smaller than 45° | $\text{tang. } A = \text{tang. } A$ | between .1 and 1 |
| B | | $\text{cot. } A = \frac{1}{\text{tang. } A}$ | between 10 and 1 |
| C | For angles A greater than 45° | $\text{tang. } A = \frac{1}{\text{tang. } (90° - A)}$ | between 1 and 10 |
| D | | $\text{cot. } A = \text{tang. } (90° - A)$ | between 1 and .1 |

These formulas will best be interpreted by examples :

**Examples :**

(A) Find tang. 30°. Place indices of T in coincidence with indices of D and read directly 5774 on D under 30° on T. Since the angle is smaller than 45°, its tangent is between .1 and 1, thus, tang. 30°=.5774.

(B) Find cot. 30°.

Place 30° mark on T over index of D and read 1732 on D under index of T. Since the angle is smaller than 45°, its cotangent is between 10 and 1, thus, cot. 30°= 1.732.

(C) Find tang. 80°.

Subtract mentally 80° from 90°, gives 10°. Place 10° mark on T over index of D and read 5675 on D under index of T. Since the original angle (80°) is greater than 45°, its tangent is between 1 and 10, thus, tang. 80°=5.671.

(D) Find cot. 75°.

Subtract mentally 75° from 90°, gives 15°. Place indices of T in coincidence with indices of D and read directly 268 on D under 15° on T. Since the original angle (75°) is greater than 45°, its cotangent is between 1 and .1, thus, cot. 75°=.268.

It should be easy now for any one to perform the reverse operation, that is, finding the angle for a given tangent or cotangent, and one example corresponding to case C will be sufficient.

**Example.**—Find the angle the tangent of which is 3.4.

Since the number given is greater than 1, the angle is greater than 45°. The tangent cannot, therefore, be read directly from the coinciding scales T and D, but one of the indices of T must be brought into coincidence with 34 on D and the angle 16° 23′ read on T over the index of D. This angle subtracted from 90° gives 73° 37′, the angle sought.

Another method of reading tangents and cotangents for angles greater than 45° is to invert the slide end for end, as previously shown in connection with cube roots, and to read the tangent directly on D under the angle on T (across S).

Figs. 21 and 22 illustrate the two methods given, applied to the last example:

$$\text{tang. } 73° 37' = \frac{1}{\text{tang. } 16° 23'} = 3.4.$$

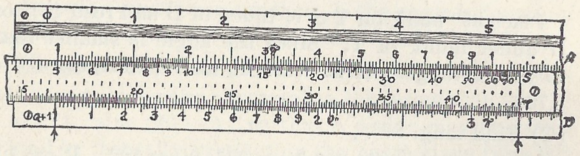


FIG. 21.

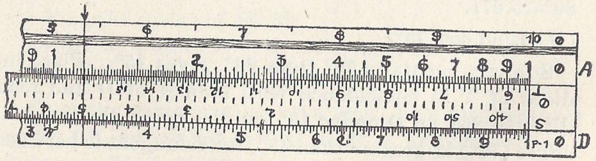


FIG. 22.



## XVI.—THE TRIGONOMETRIC FUNCTIONS IN CALCULATIONS.

For simply reading off the trigonometric functions, the slide rule is equal to a printed table. But its usefulness does not end there. A trigonometrical function almost always appears in calculations as a factor in a multiplication, or as a divisor or quotient in a division, and the slide rule enables one to perform such calculations without actually reading off the sine, cos., tang., or cot.

**Sines and Cosines as One of Two Factors.**—Use scales A and S the same as you would A and B in the ordinary multiplication, and observe for location of the decimal point that if the other factor F has P places, then the product has

P—2 places if found to the right of F in the same half scale.

P—1 places if found to the right of F in the other half scale.

P—1 places if found to the left of F in the preceding half scale.

P places if found to the left of F in the same half scale.

**Example.**—Given in a right angle triangle, the hypothenuse equal to 15.25″ and the angle 18°; what are the lengths of the arms c and b?

**Solution.**—c=15.25″ × sin 18°.

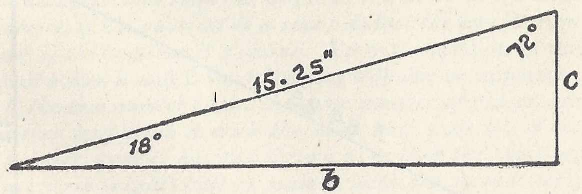



FIG. 23.

Place the left index of S under 1525 of A and read 4702 on A over 18° on S. The result appearing on the right of 1525 on the following (second half) scale A we have for the number of places

$$2-1=1, \text{ so that } c=4.702.$$

$$b=15.25\times\cos. 18°=15.25\times\sin.(90°-18°)=15.25\times\sin. 72°$$

Place the right index of S under 1525 of A and read 145 on A over 72 on S. Since this result appears to the left of 1525 on the same half scale of A, the number of places are 2 and thus:

$$b=14.5''.$$

**Sines and Cosines as Divisors.**—The division is performed as with ordinary numbers. For the location of the decimal point observe that, if the dividend D has P places, then the Quotient has

P—2 places if found to the left of D in the same half scale.

P—1 places if found to the left of D in the preceding half scale.

P—1 places if found to the right of D in the following half scale.

P places if found to the right of D in the same half scale.

**Example.**—Given in a right angle triangle one of the arms=33′ and its opposite angle=37° 30′, what is the length of the hypothenuse?

**Solution.**—$a=\frac{33}{\sin. 37° 30'}$

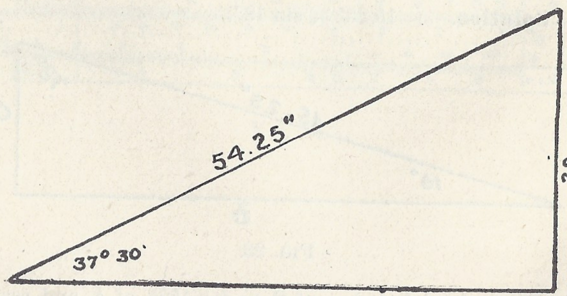



Fig. 24.



Place the 37° 30′ mark on S under 33 of scale A and read on scale A 5425 over the index of S. The result appearing to the right of 33 in the same half scale A, the number of places is 2, thus: a=54.25′.

**Sines and Cosines as Quotients.**—If in a right angled triangle the hypothenuse is a and the smaller cathete c, and the angles opposite to b and c, A and B respectively, we have

$$\sin C=\frac{c}{a}=\cos. B$$

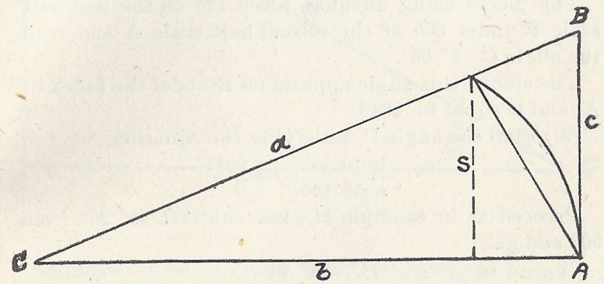



Fig. 25.

The sine or cos. respectively thus appears as a quotient. To find the angle corresponding to such a sine or cosine with the slide rule, use is made of the right hand notch in the back of the rule and the slide is used in its normal position, right side up. *Place the smaller of the given numbers read on B under the larger one, read on A, and read the angle at the notch in the back direct, if the quotient is a sine; deduct the angle from 90° if the quotient is a cosine.* From the relation of the half scales A and B the following will also be apparent: *If the two numbers have the same number of places, use either half scale A with the same half scale B; if the greater number has one figure more than the smaller, use first (right) half of scale A with the second (left) half scale B.*

**Examples.**—(1.) Find the angle C, satisfying the equation $\frac{c=5.25}{a=8.125}=\text{sine } C.$

The places of both numbers being equal, place 525 on the either half scale of B under 8125 on the same half scale of A, turn over the rule and read the angle C=40° 20′ under the notch mark on S. The sine of this angle as a decimal fraction appears on B under the right index of A, and is equal to .647.

(2.) Find the angle C, satisfying the equation

$$\frac{c=12.5}{a=365}=\text{sine C.}$$

The places being unequal, place 125 on the first half scale B under 365 of the second half scale A and read the angle C=1° 56′.

The sine of this angle appears on B under the index of A, and is equal to .0343.

(3.) Find the angle C, satisfying the equation

$$\frac{b=5.25}{a=8.125}=\text{cos. C.}$$

Proceed as in example (1), but subtract 40° 20′ from 90° and get

C=49° 40′.

### Tangents and Cotangents as One of Two Factors, or as Divisors.

With the help of formulas A, B, C and D, given on page 37, such calculations are performed by one of the following formulas:

$$\text{Product} = \text{F (D)} \times \text{tang. B.}$$

$$\text{Quotient} = \text{F (D)} \times \frac{1}{\text{tang. B.}}$$

in which B is either equal to the angle A of the given tangent or cotangent or its complement (90°—A) according to formulas A, B, C, D. For the decimal point, observe that if the factor F (or dividend D) has N places, the Product has

N places if it appears to the left of F (D).

N—1 places if it appears to the right of F (D).

the Quotient has

N+1 places if it appears to the left of F (D),

N places if it appears to the right of F (D).



**Example.**—(1.) In a right angle triangle, given one arm b=11.5 and the adjoining angle C=36°, what is the length of the other arm?

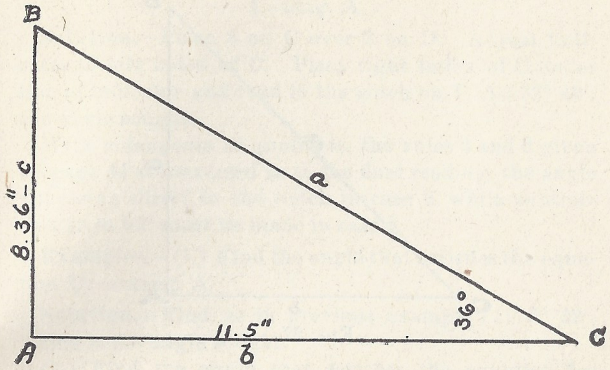



Fig. 26.

**Solution.**—c=11.5×tang. 36°. According to formula A, place left index of T over 115 of D and read 836 on D under 36° on T. Since result appears to the right of F, c=8.36.

(2.) Solve: c=8×tang. 56°.

According to formula C, tang. $56° = \frac{1}{\text{tang.}(90°-56°)} = \frac{1}{\text{tang. } 34°}$, that is, B=34°. The problem now reads

$$\text{Quotient } c = 8 \times \frac{1}{\text{tang. } 34°}.$$

Place 34° on T over 8 on D and read 1185 on D under left index of T. Since the quotient appears on the left of F, c=11.85

(3.) Solve: $c = \frac{14.5}{\text{cotang } 57°}.$

According to formula D, cotang 57°=tang. (90°—57°)= tang. 33°, thus Quotient

$c = \frac{14\ 5}{\text{tang. } 33°}$, proceed as in previous example and get c=22.3.

### Tangents and Cotangents as Quotients.

If in a right angled triangle the two arms b and c are given, we have for the angles

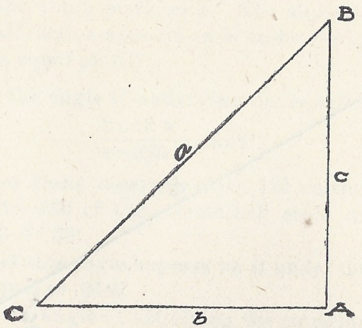



Fig. 27.

$$\text{tang. } B = \frac{b}{c} = \text{cotang. } C.$$

$$\text{tang. } C = \frac{c}{b} = \text{cotang. } B.$$

The tang. or cotang. respectively thus appears as a quotient. To find the angle whose tang. or cotang. is thus given as a quotient, use is made of the left hand notch in the back of the rule, according to the following directions:

*1. Always place the smaller number of the given fraction on C over tne larger one on D.*

*2. When the slide protrudes beyond the rule to the left, turn over the rule and read on T the angle above the mark in the left notch, subtract this angle from 90°, the difference is the angle sought.*

*3. If the slide protrudes beyond the rule to the right, run the indicator to the left index of the slide, then place the right index of the slide to the indicator mark and finally read off on T over the mark in the notch the angle sought.*

**Example.**—(1.) Find the angle that satisfies the equation $\frac{20}{3}$=tang. C.



**Solution.**—Place 3 on C over 2 (20) on D and read angle 8° 32′ in the notch. This subtracted from 90° gives 81° 28′, the angle sought.

(2.) Find the angle that satisfies the equation

$$\frac{2}{3}=\text{tang. } A.$$

**Solution.**—Place 2 on C over 3 on D. Adjust indicator to left index of C. Place right index of C under line of indicator and read in the notch on T, A=33° 42′, the angle sought.

With cotangents as quotients, the rules 2 and 3 given on page 44 are reversed as to the final reading, the angle appearing direct in the notch in case 2, while subtraction from 90° must be made in case 3.

**Examples.**—(1.) Find the angle that satisfies the equation $\frac{20}{3}$=cotang. A.

**Solution.**—Find, as in previous example (1), 8° 32′, which is the angle sought.

(2.) Find the angle that satisfies the equation $\frac{2}{3}$= cotang. A.

**Solution.**—Proceed as in previous example (2), finding angle 33° 42′, which subtracted from 90° gives 56° 18′, the angle sought.

## XVII.—TANGENTS SMALLER THAN .1. SINES SMALLER THAN .01.

The scale T starts with 5° 44′, the scale S with 34′; to get the tangent of smaller angles, advantage is taken of the fact that for small angles the sines and tangents are so little different that they may be taken for one another. Thus the tangents of angles smaller than 5° 44′ and greater than 34′ are read on the S scale as if they were sines, but it must be remembered that such tangents are smaller than .1.

**Examples.**—(1.) Tang. 3° 20′=x.

Read over 3° 20′ on S, 582 on A, and as this must be smaller than .1

x=.0582.

(2.) Find the angle whose tangent is =.026.

Read under 26 of first half scale A, 1° 29′ on S.

Since with the angle of 34′ also the scale S gives out sines and tangents of angles, smaller even than that are found from the fact that, as demonstrated in diagram below, the difference between Sine S and Tangent T is so insignificant that the value of the arc b as a medium renders the most approximate value obtainable on the Slide Rule.

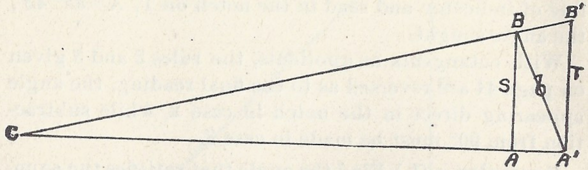



FIG. 28.

On Scale C and D will be found marks $\rho''$ and $\rho'$ at 206264 and 3438 respectively. As the arc of 1″ is equal to $\frac{1}{3438}$ and the arc of $1''=\frac{1}{206264}$, the sines and tangents of small angles are obtained by divisions with 3438 and 206264, using the above mentioned marks $\rho''$ and $\rho''$, which represents their graphical values.

The following table will be found useful:

| | | |
|---|---|---|
| Sine or Tangent of 1″ | cyphers behind the decimal point | 6 |
| " " 3″ | | 5 |
| " " 21″ | | 4 |
| " " 3′27″ | | 3 |
| " " 34′ 23″ | | 2 |

This table meaning that between sine 21″ and sine 3′ 27″ the sines have 3 cyphers behind the decimal point.

**Example.**—(1.) Sine 23′=x.

**Solution.**—Place mark $\rho'$ on C over 23 on D and read 67 on D under index of C. Since 23′ is greater than 3′ 27″ and smaller than 34′ 23″ the sine has 2 cyphers, thus:

x=.0067.

(2.) Sine 27″=x.

**Solution.**—Place mark $\rho''$ on C over 27 on D and read 1309 on D. Since 27″ is greater than 21″ and smaller than 3′ 27″

x=.0001309.



(3.) Find the angle whose tangent is equal to .00352.

**Solution.**—The tangent has two cyphers, its angle is, therefore, between 3′ 27″ and 34′ 23″. Multiply the given number by $\rho'$, by placing the right index of C over 352 on D and read 12.2 on D under $\rho''$ on C.

12.2=12° 12′.

## XVIII.—COMPOUND CALCULATIONS.

As demonstrated in chapter on Continued Multiplication and Division, consecutive multiplications and divisions can be performed without reading intermediate results, but the ultimate result only. Such was the case, also, in finding angles direct from sines and tangents given as quotients. Below we give a few more examples of such "short cuts":

**Circumference of a Circle.**—Two ways have been given previously to speedily find the circumference of a circle, using either the gauge points marked $\pi$ on the scale or by proportions. We refer to these again simply for completeness.

**The radius of a circle, the circumference of which is equal to the perimeter of a square of side A and *vice versa.*** The formula by which this radius is found is

$$r=\frac{2}{\pi}a=.636a. \qquad a=\frac{2}{.636}$$

With the help of gauge points $\rho''$ on scales C and D, this resolves itself into a simple multiplication or division respectively.

**The Area of a Circle from a given Diameter and *vice versa.*** Formula is: Square the diameter and multiply by .7854. On scales A and B find guage points corresponding to .7854; (1.) Set the right index of scale B under 7854 of A, run the indicator to the given diameter on C and read under the same indicator mark the area on A.

**Example.**—(1.) Find the area of a circle 2.5″ in diameter. The solution $a=4.91$ is given in the diagram.

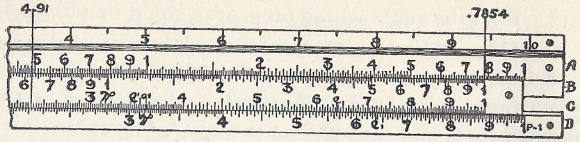



FIG. 29.

(2.) Given the area of a circle, $a=36\square''$, find its diameter. With the rule in the same setting as above, place indicator to 36 on second A scale and by means of the indicator on scale D read 6.77 on C scale.

In most cases it will be obvious on which half scale of A to place the indicator in finding the radius of a circular area. But there may be doubts, and those problems are, therefore, generally solved with the help of the gauge point marked $C$, which corresponds to $\sqrt{\frac{1}{.7854}}=1.128$. The use of this point is easily understood by the formula

$$d=\sqrt{a}\times\sqrt{\frac{1}{.7854}}=C\sqrt{a}.$$

According to it, one finds under $a$ on scale A its root on scale D and multiplies same with $C$ on scale C, reading the diameter sought on D. There is no error possible in this case as to the decimal point. For instance, take the last example, $a=36$, having 2 places its root must be sought under the second A scale, that is, one must place the indicator on 36 on the second A scale according to rule given in Chapter X. The example is illustrated in Fig. 30 below:

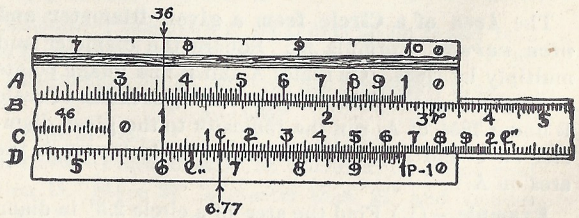



FIG. 30.



Even this discrimination between the first and second half scale A can be avoided by the use of another gauge point, $C^1=\sqrt{\frac{10}{.7854}}=3.567$ marked on scale C. The use of the two points C and $C^1$ is now obvious:

Always use for $a$ the first half scale A. If the number of places of $a$ are even use point $C^1$, if odd use point $C$. Thus for the above example $a=36\square''$, place left index of slide under 36 of first half scale A and read D=6.77 under point $C$ on scale C, because the number of places in $a$ is even.

**Example.**—Given the area of a circle, $a=123\square''$, sought the diameter.

**Solution.**—Place index of slide under 123 of first half scale A and read $d=12.55$ under $C$ on scale C, because the number of places in $a$ is odd.

**The area of a square from the area of its circumscribed circle and *vice versa.***

The formulas for these problems are:

$$a \text{ square}=a \text{ circle}\times\frac{2}{\pi}=a_c\times.636=a_c\times\rho''$$

$$a_c=\frac{a_s}{\rho''}$$

Here again the gauge point $\rho''$ comes in handy as a factor or divisor.

**The Volume of a Cylinder from its Diameter and Length.**—The formula for this is $v=.7854\,d^2\times b$, which after due transformation may be written as the proportion

$$\frac{\left(\sqrt{\frac{1}{.7854}}\right)^2}{d^2}=\frac{b}{v}$$

In the constant we recognize the gauge points $C$ and $C'$. For the slide rule formula this reads:

$$\begin{array}{r|c|c|l} \text{C scale} & C \text{ or } C' & v & \text{A scale} \\ \text{D scale} & d & b & \text{B scale} \end{array}$$

that is, put $C$ on C scale over $d$ on D scale and read the volume $v$ sought on A scale over $b$ on B scale.

**Example.**—(1.) Given of a cylinder the diameter $d$= 2.3″ and the length=7.25″, find the volume.

**Solution.**—Place $C$ on C over 2.3 on D and read 30.1□″ on A over 7.25 on B. Either point $C$ or $C'$, may be used.

(2.) The cubic contents of a gallon are equal to the volume of a cylinder 6″ high and 7″ in diameter; how many cubic inches in a gallon?

**Solution.**—Place $C'$ on C over 7 on D and read 231□″ on A over 6 on B (Point $C'$ to be used as in using $C$, the 6 on either B scale falls outside the rule).

## XIX.—THE "TRIPLEX" SLIDE RULE.

The "triplex" slide rule is a modification of the Mannheim rule, resembling the latter in construction and appearance, but differing from it by having six scales instead of four, and by a unique grouping of these scales. As will be seen from the illustration, Fig. 31, the A scale of the ordinary rule is placed as an additional scale on the top edge of the stock, and in its place is put a scale equal to the Mannheim C and D scales. The Mannheim B scale is replaced by an inverted C or D scale. The old C and D scales remain the same. On the bottom edge of the stock there is a scale not found on the Mannheim, and divided up in three equal parts. Thus there are four single scales—of which one is inverted, one double scale, and one triple scale. On the back of the slide will be found the same familiar S, T, and L scales.

The arrangement of scales described above, while permitting the performance of all the operations treated in the foregoing chapters in relation to the Mannheim rule, offers several advantages, of which those derived from the additional triple scale are the most valuable.

The figures on the triple scale being the cubes of the figures on the scale just above it (the old D scale) one can read off cubes and cube roots by just running the indicator across these two scales, the careful moving of the slide as necessary with the Mannheim rule being obviated. The only thing to be observed in reading cube



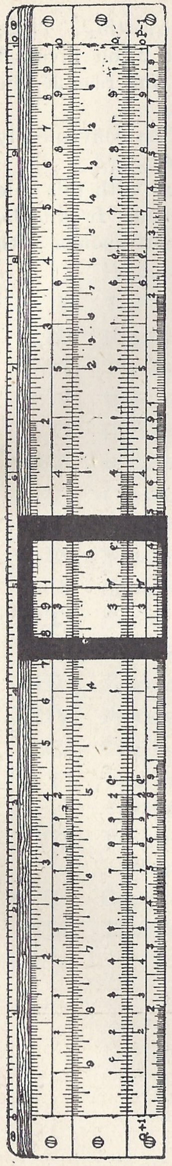


Fig. 31.

roots is, on which of the three sections of the triple scale to place the mark of the indicator. For this, one has only to remember that if the ultimate group of the radicand has one figure, the first section must be used; if it has two figures use the second section, and if it has three the third. Thus one finds the cube root of 8 on the D scale over the figure 8=2 in the first section of the triple scale, the cube root of 27=3 over 27 in the second section, and the cube root of 125=5 over 125 in the third section.

The Triple scale permits also to read such roots as

$$x=\sqrt{y^3},\quad y=\sqrt[3]{x^2}$$

in which x stands for the figures on the triple scale and y for those on the topmost scale. This will easily be understood by considering that the spaces between, like figures on these scales, are as 3 to 2.

It will also be easily understood that sixth powers can be as readily read as the fourths on the Mannheim.

The inverted slide scale is useful in solving inverted proportions. Using it in conjunction with the adjoining single rule scale for multiplication, and using the two adjoining lower scales of slide and rule for division, the result will always be found by making the two given figures coincide, beneath one of the indexes, a fact that proves often convenient.

The arrangement of the four middle single scales also permits the multiplication of three numbers in one setting. Thus, for instance, to multiply 2×3×6 set 2 on inverted scale under 3 on adjoining rule scale and read 36, the result, on D scale under 6 on C scale.

In like manner the arrangement often conveniently permits consecutive division with one setting.

The trignometric functions are treated with this rule in the same manner as with the Mannheim. It is to be observed, however, that the S scale corresponds with the outermost rule scale.



## CONTENTS.

# "PRECISION" SLIDE RULE.

**Improved Construction.**

**Mannheim Style.**

**WHITE FACINGS.**

Each Rule with Indicator, Case and Book of Directions.

| | | | |
|---|---|---|---|
| **2509** | **5 inch,** with single line Indicator..... | each, | $3.50 |
| **2562** | **8 inch,** with single line Indicator..... | " | 4.00 |
| **2564** | **8 inch,** with broken line Indicator.... | " | 4.25 |
| **2574** | **10** inch, with single line Indicator.... | " | 4.25 |
| **2576** | **10** inch, with 2 line Indicator, set to ratio of 1:12.................... | " | 4.25 |
| **2578** | **10 inch,** with broken line Indicator... | " | 4.50 |
| **2580** | **15** inch, with single line Indicator.... | " | 9.50 |
| **2622** | **20** inch, with single line Indicator.... | " | 12.50 |
| **2623** | **20** inch, with 2 line Indicator, set to ratio of 1 : 7854.................... | " | 12.50 |

## "Rival" Slide Rule.

**2711** **"Rival"** Slide Rule, 10 in., mahogany stock, celluloid facings, light construction, with Indicator.............each, $3.00

## "Stadia" Slide Rule.

Contains besides the regular scales on the **"Precision"** Slide Rule, additional scales for calculating stadia readings.

**2713** **"Stadia"** Slide Rule, 10 in., built up mahogany stock, white facings, improved construction, in case.........each, $6.50

## The "Triplex" Slide Rule.

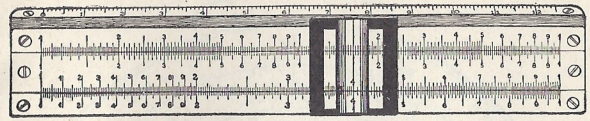
A modification of the Mannheim Slide Rule, simplifying its use when it is employed in solving complex arithmetical and trigonometrical computations.

**2832** **"Triplex"** Slide Rule, 10 in., built up mahogany stock, white facings, glazed aluminum indicator, in case..............each, $6.50



## The "Midget" Slide Rule.

**White Facings.**

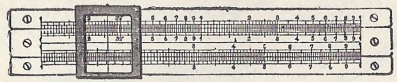


2715.

The ultimate subdivisions of the **"Midget"** are as fine as those on the regular 10 in. rule, and by means of a powerful, yet compact and convenient magnifying glass, their value is easily ascertained with the same percentage of accuracy as can be obtained by the 10 inch rules.

**2715** **"Midget"** Slide Rule, 5 in. long, built up mahogany stock, white facings, engine divided, improved construction, with magnifier, in sewed leather case with clasp and directions..........................each, $4.50

## "Vest Pocket" Slide Rule.



2717

**2717** **"Vest Pocket"** Slide Rule, 5 in., thin mahogany stock, celluloid facings, narrow for pocket use, with glazed aluminum indicator, case and directions........each, 2.25

## Nestler's Precision Slide Rule.

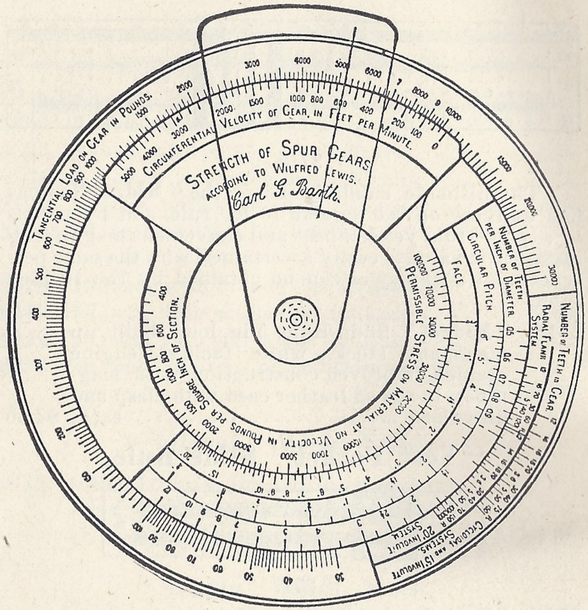
The feature of this Slide Rule is that it is really a 20 inch Slide Rule in a 10 inch length; giving all the fine subdivisions of a 20 inch Rule in a length of 10 inches.

**2761** Nestler's Precision Slide Rule, in case, each, $8.00

**2893** **The Charpentier Circular Slide Rule** was designed for vest pocket use. It consists of a German silver and a brass disc, 2⅜ inches diameter.....................each, 5.00

**2894** **The Boucher Watch Form Slide Rule** is constructed in the form of a stem-winding double-face-watch..................each, 8.50

## Barth's Gear Slide Rule.




This instrument embodies the results of Lewis' well known investigations of the strength of the teeth of gear wheels, which has been accepted as a standard by nearly the whole engineering world. It contains on its three discs all the factors entering into any problem dealing with the strength of gear teeth, and accordingly does away with all reference tables and diagrams; and it is so simple, that but little practice is needed in its use. No ambitious machine designer can afford to be without one.

**2834** Complete, with set of instructions, in box..............................each, $2.00

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Chauncey Holt, Printer, 17-27 Vandewater St., N. Y.