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A history of slide rules for blackbody radiation computations

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ABSTRACT

During the Second World War the importance of utilizing detection devices capable of operating in the infrared portion of the electromagnetic spectrum was firmly established. Up until that time, laboriously constructed tables for blackbody radiation needed to be used in calculations involving the amount of radiation radiated within a given spectral region or for other related radiometric quantities. To rapidly achieve reasonably accurate calculations of such radiometric quantities, a blackbody radiation calculator was devised in slide rule form first in Germany in 1944 and soon after in England and the United States. In the immediate decades after its introduction, the radiation slide rule was widely adopted and recognized as a useful and important tool for engineers and scientists working in the infrared field. It reached its pinnacle in the United States in 1970 in a rule introduced by Electro Optical Industries, Inc. With the onset in the latter half of the 1970s of affordable, hand-held electronic calculators, the impending demise of the radiation slide rule was evident. No longer the calculational device of choice, the radiation slide rule all but disappeared within a few short years. Although today blackbody radiation calculations can be readily accomplished using anything from a programmable pocket calculator upwards, with each device making use of a wide variety of numerical approximations to the integral of Planck's function, radiation slide rules were in the early decades of infrared technology the definitive "workhorse" for those involved in infrared systems design and engineering. This paper presents a historical development of radiation slide rules with many versions being discussed.

Keywords: Blackbody radiation, slide rules, history, Planck, radiometry

1. INTRODUCTION

When Planck introduced his equation for computing blackbody radiation in 1900, researchers were generally elated to finally have a mathematical formulation for the radiation emitted from a blackbody while being somewhat frustrated by the tedious computations necessary to compute the emission within a given spectral range. It was quickly recognized that direct integration of Planck's equation over any arbitrary spectral region was impossible. Prior to Planck's equation, Wilhelm Wien in 1896 developed an approximation that was valid only for the high-frequency portion of the spectrum. In 1905, Lord Rayleigh and Sir James Jeans derived an approximation suitable for the low-frequency portion of the spectrum. Between these two limits is a part of the spectrum that is generally of greatest interest, but no simple expression for the integration of Planck's equation within a finite spectral band exists. It was soon realized that an infinite series expansion, which involves an exponential term, could be used but it often required many terms to achieve tolerable accuracy, particularly when its argument is small (high temperature and/or low frequency regime). Herman Zanstra[1] credits Peter Debye[2] with developing in 1912 the binomial series expansion for the integral of Planck's equation within a finite spectral band. Unlike the infinite series expansion, the binomial series expansion has the advantage that for small values of the argument convergence can be obtained using only the first few terms in the expansion. Nevertheless, during the first half of the twentieth century computation of the radiation emission of a blackbody over a specific spectral band required a burdensome effort for the human computers of the day.

Although the first infrared system was most likely developed by Theodore Case[3] in 1917, and was classified Top Secret by the US Navy for ship-to-ship communications with funding ceasing in 1919, there appeared to have been little commercial or military interest in infrared systems until the mid-1930s when the drums of war began to distantly play once more. As part of his graduate work in Germany, Edgar Kutzscher[4] discovered several classes of lead salt materials could be used for infrared detection. His discoveries lead to a variety of German military applications. About the same

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time, researchers in both England and the US were pursuing similar development of infrared detectors and applications. At that time, for blackbody radiation computations, these researchers and engineers had to rely primarily upon laboriously prepared tables.

A modest chronicle by Lowan and Blanch[5] presents references and comments about the early tables and graphs of blackbody radiation. Apparently the first tables were produced by Frehafer and Snow at the US National Bureau of Standards in 1925,[6] and had an accuracy of four significant figures. Other tables were published by Fabry[7] in 1927, Holladay[8] in 1928, Fowle[9] in 1929, Skogland[10] in 1929, Jahnke and Emde[11] in 1933, Yamauti and Okamatu[12] in 1936, Moon[13] in 1937, and Miduno[14] in 1938. Only Miduno, Moon, and the Lowan and Blanch tables had increased accuracy to up to five significant figures. Interest in this subject was clearly concentrated in Germany, the US, France, and Japan. As improvements in the physical constants occurred, so did the need to refine the tables.

Instrumentation and systems technology were also evolving and were a driver in needing increasingly better computation of the blackbody functions. Early systems in the 1950s and 1960s required perhaps 1% accuracy and the breadth of infrared systems applications was steadily increasing. In addition to ground-to-air and air-to-air missiles, there were applications for infrared gun sights, surveillance devices, space sensors, etc. This increased interest resulted in a number of new tables being produced and often for specific purposes. Examples of such tables include LaFara[15] in 1955, Pivovonsky and Nagel,[16] and Czerny and Walther,[17] both in 1961, Walker[18] in 1962, Bowen[19] in 1963, Pisa[20] in 1964, and Gebel[21] in 1969.

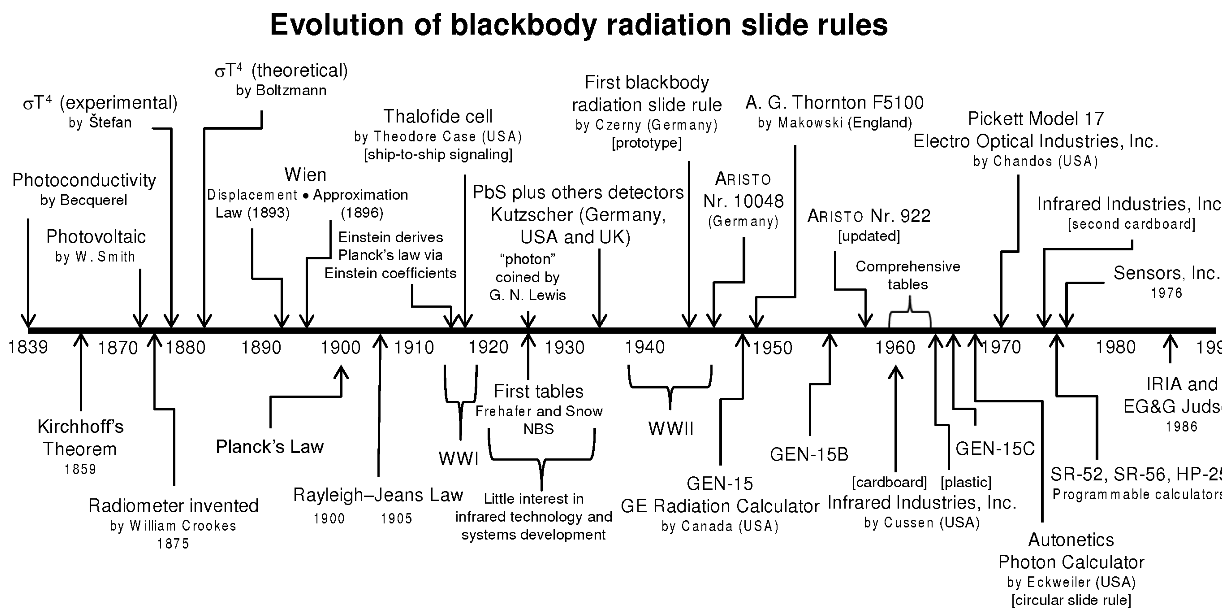

Figure 1. A timeline showing the evolution of blackbody radiation slide rules in the context of other historically important events relating to blackbody radiation.

During the latter part of the Second World War, both Germany and the Allied Forces were utilizing infrared systems as part of their war-fighting equipment. Although analog computers were available by then, digital computers were not. No mention has been uncovered that indicates the use of an analog computer to compute blackbody emissions. Digital computers were extraordinarily rare throughout the 1950s with users having great difficulty scheduling time on them. During the 1960s, mainframe computers and minicomputers began to proliferate; however, turnaround time was long. One author (RBJ) recalls while working at Texas Instruments in the late 1960s, turnaround time was at least a day and often three days! Motivated by means to rapidly perform the numerous radiometric computations the German military required of his organization most likely led the German experimental physicist Marianus Czerny to design and construct the first blackbody slide rule in 1944. After the Second World War ended, a variety of infrared slide rules began to appear and were welcomed by scientists and engineers worldwide. The computational accuracy of the various slide rules varied depending upon the temperature, wavelength, or type of computation involved, but it was not unrealistic to achieve accuracies of 1%

or better from such devices. Figure 1 presents a timeline of the evolution of blackbody radiation slide rules with a number of related historical events beginning in 1839 and continuing until the introduction of programmable hand-held calculators that resulted in the sudden demise of these remarkable devices used for thermal radiation calculations. Not only were the new programmable calculators quick and affordable, they were able to achieve computational accuracies of 10^{-9}.[22] The various radiation slide rules that were devised and commercially manufactured in Germany, England, and the US are now only available in the used marketplace. Following a brief review of blackbody radiation, a rather comprehensive discussion of the origin and operation of a selection of the more widely used radiation slide rules is presented in section 3.

2. REVIEW OF BLACKBODY RADIATION

All objects emit radiation as a result of their temperature. Known as thermal radiation the body that radiates the greatest amount compared to all other bodies at the same temperature is referred to as a *blackbody*. Knowledge of such a body allows it to serve as the ideal theoretical standard against which all real radiating bodies can be compared. Understanding the nature of blackbody radiation was one of nineteenth century physics crowning achievements and paved the way to the quantum revolution at the beginning of the twentieth century. We begin by briefly reviewing the laws of thermal radiation needed in order to understand the scales found on the various radiation slide rules we intend to consider in this paper.

While having access to a radiation slide rule was not essential, it would quickly become an indispensable computational aid to those working principally in the infrared portion of the electromagnetic spectrum. In building devices to detect radiation in this portion of the spectrum the type of detector used can be divided into two broad classes depending on the physical mechanism involved in the detection process. The first are the so-called *thermal* detectors. Here a heating effect caused by the incident radiation results in a variation in some physical parameter (usually electrical) of the detector with temperature. The second are the so-called *photon* detectors. Here there is a direct interaction between the incident photons and the electrons of the detector. In the former case the detector response will be proportional to the energy absorbed while in the latter case it is proportional to the number of photons absorbed. Connecting particular units in a natural way to the type of detector used turns out to be useful when discussing the performance of detectors, an important consideration in the design of any radiation slide rule. *Radiometric* quantities, that is those quantities used to describe radiant energy, are based on the joule (or watt) and is the natural unit of choice for thermal detectors (so-called "energetic" units). *Actinometric* quantities, that is those quantities related to the measure of photons, are more appropriate for photon detectors (so-called "photonic" units). Since both types of quantities are found on the various radiation slide rules we intend to consider, each is briefly reviewed.

The occurrence of very hot objects changing color as their temperature increases is familiar to most and is embodied in the well known *Stefan–Boltzmann law*. It states that the total energy emitted from the surface of a blackbody at temperature T per unit time per unit area in all directions into the half space above the surface is proportional to the fourth power of the temperature. Mathematically

$$M(T) = \sigma T^4. \tag{1}$$

Here M is the total *emittance* and is measured in units of watts per square meter [W m^{-2}] while σ ($5.670\,373 \times 10^{-8}$ W m^{-2} K^{-4}) is the Stefan–Boltzmann constant. The law is named in honour of Jožef Štefan who deduced the law empirically in 1879 and Ludwig Boltzmann who derived the result theoretically some five years later using arguments based on thermodynamics.

In addition to knowing the total energy radiated by a blackbody at a given temperature, understanding how this energy is distributed throughout the spectrum as a function of wavelength is equally important. By the late nineteenth century it was experimentally known that the radiation emitted from a blackbody was spread continuously over a singly peaked spectrum consisting of all wavelengths. However finding the mathematical form that described this spectral distribution became one of the great unsolved problems facing physics at the turn of the twentieth century. In 1893, using thermodynamic arguments together with the Doppler effect, Wilhelm Wien was able to deduce theoretically the general form the equation for the spectral distribution of radiation from a blackbody must obey in the form of his displacement law. The special case of this general law is often cited and relates to the peak value found in the spectral distribution curve for a blackbody as a function of wavelength. It states that

$$\lambda_{\max} T = b. \tag{2}$$

Here b ($2.897\,7721 \times 10^{-3}$ m K) is a constant and in this special form the law is known simply as *Wien's displacement law*.

To find the form for the spectral function, one lets the quantity $M(\lambda, T)\, d\lambda$ denote the amount of energy within a part of the spectrum given by the spectral range λ to $\lambda + d\lambda$ radiated by a blackbody into a hemispherical envelope in space per unit time per unit area. In 1900, Max Planck showed the *spectral emittance* $M(\lambda, T)$ in the linear wavelength representation is given by his now famous law

$$M(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 \left[\exp\left(\frac{hc}{k_B \lambda T}\right) - 1\right]}. \tag{3}$$

Here the fundamental constants c ($2.997\,924\,58 \times 10^8$ m s^{-1}), h ($6.626\,069 \times 10^{-34}$ J s), and k_B ($1.380\,6488 \times 10^{-23}$ J K^{-1}) are the speed of light in vacuo, Planck's constant, and Boltzmann's constant respectively. The units for spectral emittance are watts per square meter per unit wavelength interval [W m^{-2} μm^{-1}]. While the Stefan–Boltzmann and Wien displacement laws antedate Planck's formulation, both are direct consequences of the latter's law.

In applications the amount of energy radiated by a blackbody into a given spectral band is often required. Between wavelengths λ_1 and λ_2 ($\lambda_2 > \lambda_1$) it is found from

$$M_{\lambda_1 \to \lambda_2} = 2\pi hc^2 \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^5 \left[\exp\left(\frac{hc}{k_B \lambda T}\right) - 1\right]}. \tag{4}$$

A more convenient form, particularly when used as a scale on a slide rule, is the fractional amount normalized relative to the total emittance $M(T)$. Starting from properties for the definite integral, Eq. (4) can be re-written as

$$M_{\lambda_1 \to \lambda_2} = M_{0 \to \lambda_2} - M_{0 \to \lambda_1}, \tag{5}$$

where

$$M_{0 \to \lambda_*} = 2\pi hc^2 \int_0^{\lambda_*} \frac{d\lambda}{\lambda^5 \left[\exp\left(\frac{hc}{k_B \lambda T}\right) - 1\right]}. \tag{6}$$

Here λ_* is any arbitrary wavelength. Employing the substitution $x = hc/(k_B \lambda T)$, the integral appearing in Eq. (6) can be made dimensionless. When this is done one obtains

$$M_{0 \to \lambda} = \frac{15}{\pi^4} \sigma T^4 \int_z^{\infty} \frac{x^3}{e^x - 1} dx, \tag{7}$$

where $z = hc/(k_B \lambda_* T)$. Finally, as a fractional amount it is given by the ratio

$$\mathfrak{F}_{0 \to \lambda} = \frac{M_{0 \to \lambda}}{M(T)} = \frac{15}{\pi^4} \int_z^{\infty} \frac{x^3}{e^x - 1} dx. \tag{8}$$

This fractional amount is one of the most important quantities found on all radiation slide rules. As it was thought the integral appearing in Eq. (8) could not be expressed in closed form in terms of any of the known functions of mathematical physics meant its evaluation was tedious and time consuming. However, as an often required quantity, an importance reflected in the large number of approximations developed over many decades to specifically deal with this integral,[8,23–30] its reduction to a quick and simple slide rule calculation that remained reasonably accurate would no doubt come as a welcome relief.

Emittance describes the radiant flux (rate of energy transfer) leaving a surface. It however gives no information about the directional dependence of the emitted radiation. For this one introduces the concept of *radiance* L. For a perfectly diffuse (Lambertian) radiator such as a blackbody, when the total radiance is summed over all possible space the radiation can be emitted into (a hemispherical envelope in space above the surface of the body), the following simple relationship between the total emittance and total radiance is found

$$L(T) = \frac{1}{\pi} M(T). \tag{9}$$

As a directional quantity it represents the part of the total emittance falling into a certain section in space and has the units of watt per square meter per steradian [W m^{-2} sr^{-1}].

All radiometric quantities presented above can also be given in terms of photons. Since the energy E in joules carried by a photon depends on its wavelength and is given by

$$E = \frac{hc}{\lambda}. \tag{10}$$

Since this equation gives the energy carried per photon, it can be used to convert any spectral radiometric quantity given in energetic units into its corresponding spectral actinometric quantity given in photonic units on dividing by hc/λ. Planck's law for the spectral photon emittance is therefore

$$M_q(\lambda, T) = \frac{2\pi c}{\lambda^4 \left[\exp\left(\frac{hc}{k_B \lambda T}\right) - 1\right]}. \tag{11}$$

The symbol q appearing in the subscript will be used to denote an actinometric quantity is being considered. The units for $M_q(\lambda, T)$ are photons per second per square meter per unit wavelength interval [photon s^{-1} m^{-2} μm^{-1}]. The total photon emittance will be

$$M_q(T) = \sigma_q T^3, \tag{12}$$

and is known as the Stefan–Boltzmann law for photon emittance. The units for $M_q(T)$ are photons per second per square meter [photon s^{-1} m^{-2}] while σ_q ($1.520\,461 \times 10^{15}$ s^{-1} m^{-2} K^{-3}) is the Stefan–Boltzmann constant in photonic units. For the fractional amount equivalent to Eq. (8) in photonic units, one has

$$\mathfrak{F}_{q,0\to\lambda} = \frac{M_{q,0\to\lambda}}{M_q(T)} = \frac{1}{2\zeta(3)} \int_z^\infty \frac{x^2}{e^x - 1} dx, \tag{13}$$

where $\zeta(x)$ is the Riemann zeta function. Finally, Wien's displacement law in photonic units becomes

$$\lambda_{q,\max} T = b_q. \tag{14}$$

Here b_q ($3.669\,7030 \times 10^{-3}$ m K) is the corresponding Wien displacement law constant in photonic units.

Finally, as all real bodies encountered in practice are not perfect blackbodies, the *emissivity* ε is defined as the ratio of the total emittance of a body to that of a blackbody at the same temperature. It is a dimensionless quantity between zero and one and is a measure of a body's radiating (and absorbing) efficiency. A spectral emissivity ε_λ at each wavelength can also be defined. In the *greybody* approximation the spectral emissivity is independent of wavelength and in such cases all radiometric and actinometric quantities corresponding to a greybody are reduced by a constant factor equal to the body's emissivity compared to those for a blackbody. As an example, the total emittance and the total photon emittance of a greybody are given by $M(T) = \varepsilon\sigma T^4$ and $M_q(T) = \varepsilon\sigma_q T^3$ respectively.

3. PRINCIPAL SLIDE RULES USED FOR THERMAL RADIATION CALCULATIONS

Until it was displaced by the arrival of cheap electronic calculators in the mid-1970s, the slide rule was an indispensable tool of the scientist and engineer. Designed to relieve the user of the drudgery associated with having to perform largely routine computations by hand, many different types and designs had been proposed and made for a bewildering variety of purposes. These ranged from relatively standard types that could be used to multiply and divide numbers, extract square and cube roots, or determine values for the trigonometric, logarithmic, and exponential functions, to those designed and used for highly specialized purposes.

The field of thermal radiation readily lends itself to a special purpose slide rule. Calculations are particularly tedious due to the cumbersome mathematical form the laws of thermal radiation take on. One needs little convincing of the advantage gained by having access to a calculational aid capable of quickly performing many of the often repetitive calculations encountered in the field. While the need was probably recognised by many, surprisingly it was not until late 1944 in war-torn Germany that the first of the many so-called radiation slide rules appears. Prior to the availability of specialized slide rules for thermal radiation calculations, workers in the field either used one of a number of tables that existed for various blackbody radiation quantities at the time[5,6,8,10,14,31] or reverted to performing any needed calculation by hand together with possible reference to more standard tables of mathematical functions such as the exponential function.

The first of the radiation slide rules was designed by the German experiential physicist Marianus Czerny* and was of the linear type. Later radiation slide rules included linear and circular forms made of plastic, aluminium, or wood coated with a white celluloid veneer, or slide charts made of either finished cardboard or more robust plastic that were often given away as promotional items. All were capable of estimating various physical quantities associated with the radiation emitted by a blackbody at a certain temperature, with the number of quantities included reflecting the intended use for each rule. In what follows, we give a number of examples of the many radiation slide rules available from their inception right up until the late 1980s.

3.1 The *System Czerny* slide rules by Aristo

By 1944 when a hand-built prototype for a radiation slide rule first appeared in the German periodical *Physikalische Zeitschrift*[34] Marianus Czerny was already well known for his work in the infrared. In his paper he writes it was due to the inability of not being able to integrate Planck's law, namely Eq. (4), in closed form over an arbitrary spectral wavelength range that prompted him to design a slide rule for the sole purpose of solving this particular problem.†. It was the most important scale to appear on his rule. In finding the various mark locations that would make up this scale the evaluation was performed by hand by himself together with the assistance of a certain Mr Kurt Schäfer who was presumably either one of Czerny's students or technical assistants who worked for him at the time in his laboratory at the Johann Wolfgang Goethe–Universität in Frankfurt am Main.

Compared to radiation slide rules that were to come, Czerny's rule is a relatively simple affair. It contains just four scales and two gauge marks. It was a single-sided linear slide rule of the closed-frame construction type and measured about eight inches long. Initially his slide rule came without a cursor and it seems a small number of the rules were hand built and used by his students and staff in his infrared laboratory at Frankfurt. A few years later his design served as a prototype for a rule produced by ARISTO, a large German manufacture of slide rules at the time. It was designated the "System Czerny" after its inventor, was made of plastic, measured about 30 cm long, and came with a fixed cursor. Commercially it was known as the ARISTO Nr. 10048 – *Rechenschieber für Temperaturstrahlung*. It is thought the rule became available in Germany shortly after the end of the Second World War. By the late 1940s it was available in the US, being obtainable from George Haas who at the time worked at the Engineering Research and Development Laboratories at Fort Belvoir in Virginia.[39] A photograph of the front face of the 10048 is shown in Fig. 2.

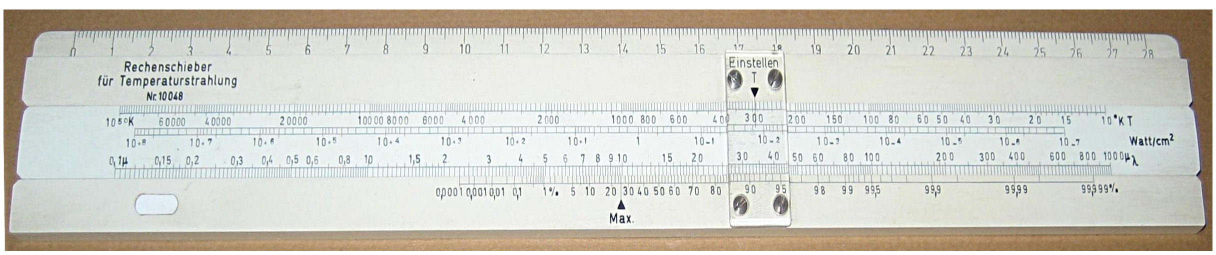

Figure 2. Front face of the ARISTO Nr. 10048 – Rechenschieber für Temperaturstrahlung from the late 1940s.

Turning to the scales that appeared on the rule, running along the top of the bottom stock was the scale for the fractional amounts found numerically from Eq. (8). These were expressed as a percentage and ran from 0.001 to 99.999%. The gauge

*Professor Dr. phil. Dr. rer. nat. h. c. MARIANUS CZERNY was born on the 17 February 1896 in Breslau and died on 10 September 1985 in Munich. He received his *Doctor philosophiae* (Dr. phil.) from the Universität Berlin in 1923 for a thesis on the so-called "Reststrahlen" (residual ray) method. The shear breadth of his scientific activities, much of it pioneering experimental work in the infrared, was the result of a sixty year career which spanned well into his retirement. For example, in 1929 he developed what he called "evaporographie", a photographic technique for making heat radiation visible while a year later working with A. Francis Turner he developed their now famous *Czerny–Turner* monochromator. Five years after his retirement in 1966 he was awarded a *Doctor rerum naturalium honoris causa* (Dr. rer. nat. h. c., literally "honorary doctor of the natural things"), an honorary doctorate by his former institution the Johann Wolfgang Goethe-Universität. A short summary of his most important scientific work in English is given by Genzel, Martienssen, and Mueser[32] while a more extensive biography of his life and work in German is given by Wiesbaden.[33]

†While it is not possible to find a closed-form expression to the integral of Planck's law over an arbitrary spectral wavelength range in terms of elementary transcendental functions[35] it is still possible to write the integral in closed form in terms of the special function known as the polylogarithm[36–38] These are all later developments not known to Czerny at the time he tackled the problem in 1944.

mark “Max” on the bottom stock gave the fractional amount when the interval from zero up to the peak wavelength in the spectral curve as given by Wien’s displacement law is considered. In value it is very close to 25%. On the slide a logarithmic temperature scale T, measured in Kelvin (K), from 10 to 10^5 K ran along the top and a reverse logarithmic scale for the wavelength λ, measured in micrometers (μm, though on the rule the unit appears simply as μ), from 0.1 to 1000 μm ran along the bottom. Running through the middle of the slide is a scale for the total radiance $L(T)$. In this regard Czerny was unusual. Later radiation slide rules that appeared in England and the US always used the total emittance $M(T)$ rather than the total radiance $L(T)$. The scale ran from 10^{-7} to 10^8 and was measured in watts per square centimeter per steradian [W cm^2 sr^{-1}].

Labelled “T einstellen” (temperature setting) on the top stock is the second gauge mark. In operation the slide was adjusted to the desired temperature by aligning the value of the temperature with the upper gauge mark. The total radiance could then be read off from the scale appearing in the middle of the slide. At the selected temperature the wavelength where the spectral radiation curve peaks could also be read off directly from the bottom gauge mark. Lastly, using the wavelength scale on the bottom of the slide in conjunction with the adjacent percentage scale on the bottom stock allows the fraction of the total radiance within the wavelength range from zero up to some arbitrary value for the wavelength λ to be determined. A centimeter scale from 0 to 28 is marked along the top of the rule. The addition of this scale however had nothing to do with the operation of the slide rule and merely allowed it to double as a simple ruler.

An updated version of the 10048 appeared in the late 1950s under the new model number of 922. It is likely it became available either in late 1957 or early 1958 and was known as the ARISTO Nr. 922 – *Rechenstab für Temperaturstrahlung*. The number of scales and gauge marks remained unchanged, however unlike the 10048, the 922 came with an adjustable cursor and its construction was now of the open-frame type. In operation the 922 functioned in exactly the same manner as its predecessor. The adjustable cursor now allowed for more accurate fractional parts to be read for any arbitrary wavelength of interest. As a further minor improvement to the rule, clear labels for each of the scales were added to the far left hand end of the rule. Vertically aligned, reading from top to bottom these were T, Q, λ, and W(z) for temperature, total radiance, wavelength, and fractional amount. Note the 922’s symbols Q and W(z) correspond to the modern day symbols of L and $\mathfrak{F}_{0\to\lambda}$ respectively. The positioning of an adjustable cursor running along the outer edges of the rule also meant the rule no longer had an uninterrupted straight edge. The former centimeter scale found on the 10048 was therefore done away with. A photograph of the front face of the 922 is shown in Fig. 3.

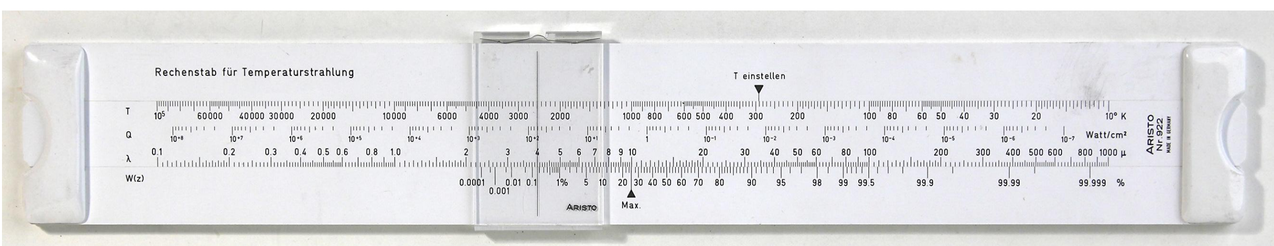

Figure 3. Front face of the ARISTO Nr. 922 – Rechenstab für Temperaturstrahlung from the late 1950s. Photograph courtesy of Bruno Ferrighi.

Outside of Germany, the two radiation slide rules made by ARISTO do not appear to have been widely known. Each rule is referred to in the first and second editions of Werner Brügel’s important early infrared text *Physik und Technik der Ultrarotstrahlung*.[40,41] Reference to it is also made in *Ullmanns Encyklopädie der technischen Chemie* of 1960,[42] a widely read reference work that dealt with all aspects of the science and technology of industrial chemistry. References to the rule in the English literature are somewhat more limited. It appears the first reference to the rule in an English language publication came in 1947 and is found in *European Scientific Notes*, an informal, bimonthly publication of the London Branch of the Office of Naval Research (ONR).[43,44] Established in 1946 to survey, assess and report on European scientific and technological activities, ONR London’s best-known output would become its European Scientific Notes. Carrying news of noteworthy developments in European scientific research its very first issue carried a piece highlighting developments Germany had made in the infrared during the war years based on recently conducted interviews with Marianus Czerny and Gerhard Hettner. Czerny’s principal areas of research were divided into five areas with the fifth entitled “Minor theoretical considerations of black-body radiation”. Here one of the minor theoretical considerations is reported as being "... a useful slide rule... for obtaining the intensity (watt/cm^2) of black-body radiation in any desired finite wave length region, once the temperature is given".[45] Some years later in 1962 in a book review for a recent collection of tables for the fractional function of Planck's radiation law we find the reviewer, John N. Howard, writing the radiation slide rule produced by Prof. Czerny "... was useful for quick calculations".[46] Richard D. Hudson in his 1969 text *Infrared System Engineering*[47] briefly mentions the Aristo 922 in a section devoted to a general discussion on the various slide rules available at the time for thermal radiation calculations. He manages to do this in two very short sentences and a footnote. While the "System Czerny" rule seemed to be used by relatively few outside of Germany, by far its most lasting impact is to be seen in the radiation slide rules it inspired in others to design and be made.

3.2 The Radiation Calculator slide charts from General Electric

The aptly named Radiation Calculator from General Electric would come to dominate a generation of engineers working in infrared systems design and development in the US. It was a rule made for the General Electric Company in 1948 based on a design by the then young engineer Alfred H. Canada.‡ The design for Canada's rule first appeared in the company's in-house journal *General Electric Review*.[39] Compared to the radiation slide rule made by Aristo in Germany, Canada's rule was a very different affair. A far greater number of scales were to be found on his rule, and in the strictest sense, Canada's slide rule was not a slide rule at all but an example of what today we would call a slide chart.

In tracing the origin of Canada's rule one finds it is firmly rooted in the rule proposed, designed, and built by Czerny. In his paper of 1948 he writes that as far as he was aware the only radiation slide rule to come before his own was the rule of Czerny's. It is known that shortly after hostilities in Europe ended, Canada went to Germany as a member of one of the many scientific reconnaissance missions from the US that entered the country at the time. His work involved assessing the technological developments Germany had made in the infrared during the intervening war years and resulted in the publication of the report, *Infrared: Its Military and Peacetime Uses* two years later in December 1947.[48] We suspect it was during one of these missions Canada first became acquainted with the radiation slide rule of Czerny that led to the design of his own rule.

The first radiation slide chart of Canada's was known as a "Radiation Slide Rule". It was designated GEN-15 and was made for the General Engineering and Consulting Laboratory, General Electric, Schenectady, New York by the prolific slide chart manufacturer Perrygraf Corp. Both the card and slide were made of finished cardboard, it was double sided while the various scales found on the rule were grouped into six panels, three on each side. On the front of the card were eight scales while the back contained a further six. There was no cursor. Many of the scales provided conversions between a number of different system of units in common use at the time. The front and back faces of the General Electric slide chart GEN-15 are shown in Fig. 4.

The top panel on the front of the slide chart was for temperature, the top scale in units of degrees Celsius (C) the bottom in Kelvin (K). Two gauge marks (heavy arrows) above and below the panel on the card were used to set the required temperature. Reversing the slide chart, the panel at the top gave the temperature in units of Fahrenheit (F) and Rankine (R). The middle panel on the front of the slide chart was for the total emittance. The top scale was in units of watts per square meter [W m^{-2}] while the bottom gave the total emittance in units of calories per square centimeter per second [cal cm^{-2} s^{-1}]. Aligned with the top and bottom of the second panel and fixed on the card a scale for the emissivity running from 0.05 to 1 appears. The slide chart could therefore be used to determine total emittances for both a blackbody ($\varepsilon = 1$) and greybodies. A gauge mark at $1/(2\pi)$ also appears on the emissivity scale. Presumably it was provided to allow for the quick conversion between total emittance and radiance of a blackbody to be made. The conversion factor given however is incorrect and instead should have been $1/\pi$ for such a purpose. The middle panel found on the reverse side of the slide chart gave the total emittance at various emissivities in units of watts per square inch [W in^{-2}] and in British thermal units per square foot per hour [Btu ft^{-2} hr^{-1}]. Further conversions for the total emittance at various emissivities in two additional system of units appear on the bottom panel on the reverse side of the slide chart. Here the top scale gave

‡ALFRED H. CANADA was born in Portland, Oregon on 14 November 1918. In 1940 he graduated with a degree in electrical engineering from Oregon State University and shortly afterwards joined his country's war effort, performing military service at the Engineering Research and Development Board at Fort Belvoir in Virginia. Much of his early work focused on military applications of the infrared and was done while he worked at the General Engineering and Consulting Laboratory at General Electric in Schenectady, New York. He later moved into engineering management before retiring in 1974. He was issued with eleven US patents between the years 1946 and 1963. As part of a self-funded retirement avocation he was involved in the design and development of large-scale solar photovoltaic generation power plants. He died in 2002.

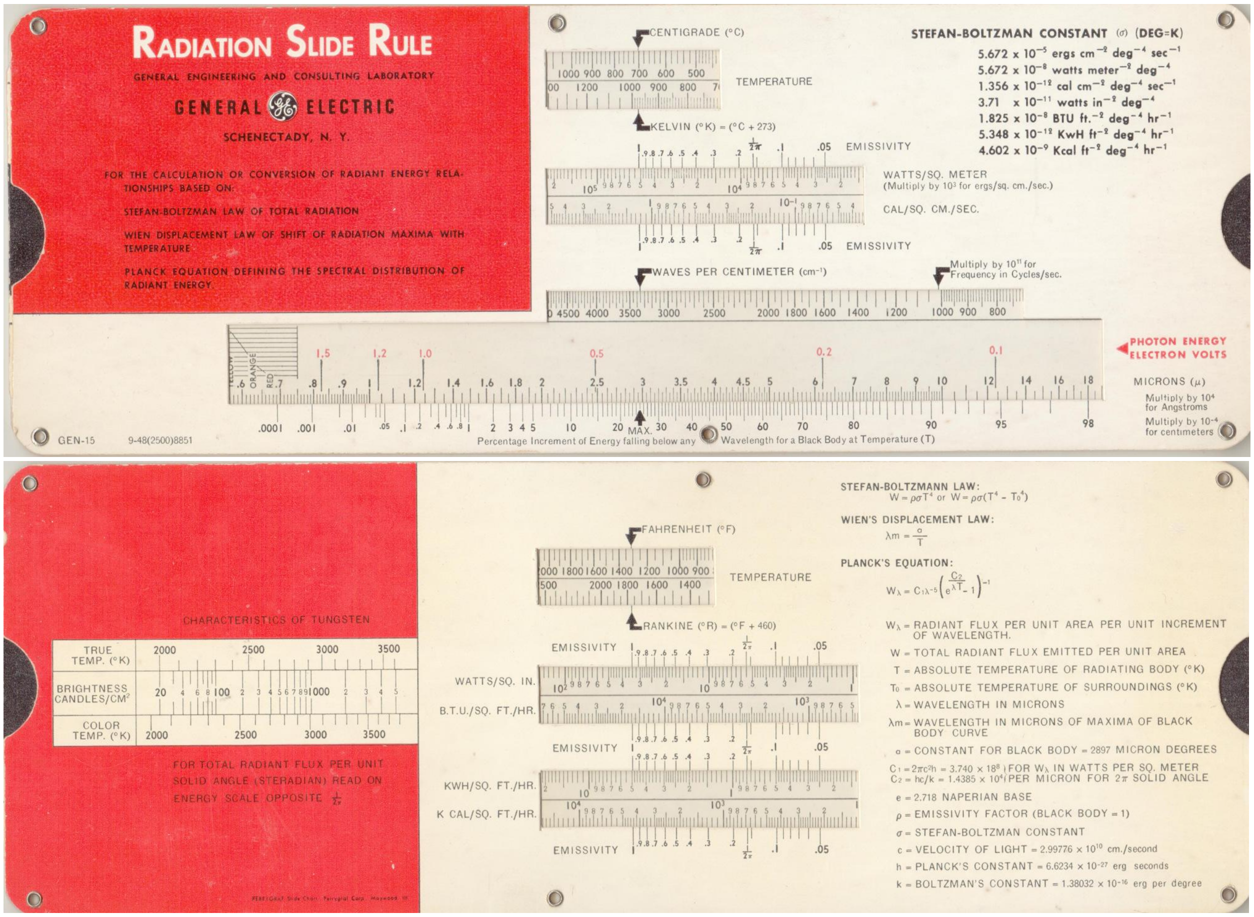

Figure 4. Front (top) and back (bottom) faces of the General Electric slide chart GEN-15.

the total emittance in units of kilowatt-hours per square feet per hour [kWh ft^{-2} hr^{-1}] while the bottom one in units of kilo-calories per square feet per hour [kcal ft^{-2} hr^{-1}].

The third panel located at the bottom of the front of the slide chart was the reason for the sliding part of the rule. Running along the bottom of the panel on the slide is a reverse logarithmic scale for the wavelength λ, measured in micrometers (μm, though on the rule itself it appears as microns, μ, as was custom at the time). Aligned with the wavelength scale but fixed to the card is a scale for the fractional amount of the total emittance radiated by a blackbody into a given spectral band from zero up to some arbitrary wavelength for a blackbody at a temperature T. It is expressed as a percentage and runs from 0.0001% to 98%. Setting the temperature in the top panel allows the fraction of the total emittance radiated by a blackbody into a given wavelength interval to be determined using the bottom panel. A gauge mark "MAX" appears on the scale running along the bottom of the card. It gives the fractional amount when the interval from zero up to a wavelength corresponding to the peak found in the spectral emittance curve is considered. Above the wavelength scale appears an additional scale in red. It gave the corresponding energy a photon has, in electron volts (eV), for a given wavelength. Finally, running along the very top of the bottom panel is a scale for the wavenumber $\tilde{\nu}$ measured in waves per centimeter [cm^{-1}].

In the hope of increasing the usefulness of the rule further, Canada included a number of other features on the rule. On the front of the slide chart in the top right hand corner appeared values for the Stefan–Boltzmann constant σ in no less than seven different system of units. Though not mentioned in his paper of 1948, the values Canada uses are those given in 1941 by Raymond Thayer Birge.[49] These were the most accurately known values for the fundamental physical constants at the time and were widely used. On the reverse side, one third of the right end of the slide chart is taken up with formulae for the Stefan–Boltzmann law, Wien's displacement law, and Planck's law. At the left end and taking up a further third of the slide chart is printed a nomogram showing specific characteristics of tungsten. Here color temperature (the temperature required by a blackbody to radiate light of a comparable hue to that of the light source) and brightness in candles per square centimeter as functions of the true temperature of the body in Kelvin are given. Below the equations for the three laws are defined fifteen different symbols for a number of different radiometric quantities and physical constants. Values for all physical constants appear on the rule. Formerly used symbols for the total and spectral emittance, W and W_λ, and the emissivity, ρ, are given. The modern-day equivalents are M, M_λ, and ε respectively.

Shortly after Canada's paper appeared, news of the rule's availability was quickly reported[50] and it was not long before others started referring to his rule in the technical literature.[51,52] The rule itself could be obtained postpaid for $0.75 from the General Electric Company, 1 River Road, Schenectady, New York. Owing to the popularity and success of the first radiation slide rule, an updated version of the slide chart was released in 1956. Its name was changed to "Radiation Calculator", the number of scales appearing on the rule compared to its predecessor was increased, and it came with a brief four page instruction leaflet that gave six examples showing how the slide chart could be used. Designated GEN-15B, its construction was also improved. It now consisted of a flexible plastic slide inside a more robust and rigid transparent vinyl sleeve that was screen printed on its inner sides. Clear panels on both the front and back of the sleeve allowed the scales on the inserted slide to be read more easily and accurately than the GEN-15.

On the front of the GEN-15B four panels with a total of eleven scales are found while on the back there appear five panels containing a further eleven scales. The top panel on the front of the slide chart contained scales for the temperature in both degrees Celsius (C) and Kelvin (K). Reversing the rule, the panel centred in the middle of the chart at the top had the two temperature scales in Fahrenheit (F) and Rankine (R). As was the case with its predecessor, conversion of temperatures among the four different temperature units by setting the temperature on one scale and reading on any of the other three was once more possible. The scale immediately below the temperature panel on the front of the slide chart and to the right was a standard C/D logarithmic scale. It was a new scale for the GEN-15B not found on the GEN-15. Presumably this scale was added in case one either misplaced or forgot to bring one's standard slide rule along for the day.

The top scale running along the middle panel of the front of the rule gave, at a particular temperature setting, the total emittance in units of watts per square centimeter [W cm^{-2}]. An emissivity scale associated with this scale running from 0.05 to 1 on the card above the panel allowed the total emittance for a greybody to be found. On the emissivity scale a new gauge mark of $1/\pi$ appears. Since the total radiance L for a blackbody is related to the total emittance M by $M = \pi L$, the scale allowed for the determination of both total emittance and total radiance and corrected the erroneous gauge mark located at $1/(2\pi)$ on the GEN-15. An emissivity equal to one gives the total emittance for a blackbody while at the gauge mark of $1/\pi$ gives the corresponding total radiance for a blackbody in units of watts per square centimeter per steradian at the particular temperature setting chosen. On the reverse side, the middle panel just below the temperature panel gave the total emittance and corresponding emissitivies in units of watts per square inch [W in^{-2}] along the top scale of the panel and in British temperature units per square foot per hour [Btu ft^{-2} hr^{-1}] along the bottom scale of the panel. It represented a reduction of three compared to the six scales found for this radiometric quantity on the GEN-15. The bottom scale found on the middle panel on the front of the slide chart gave the spectral emittance for a blackbody at the peak wavelength found in the spectral emittance curve when plotted as a function of wavelength at a given temperature. It was given in units of watts per square centimeter per unit wavelength interval [W cm^{-2} μm^{-1}]. The scale did not appear on the GEN-15 but was a very useful addition since it could be used in conjunction with one of the other new scales to calculate the spectral emittance, a quantity that could otherwise not be directly found using the slide chart.

The very long panel appearing on the front of the slide chart at its base on the GEN-15B was significantly extended compared to the one found on its predecessor. In common with the GEN-15, running along the bottom of the panel on the slide was a reverse logarithmic scale for the wavelength, measured in micrometers. Aligned with this scale was a fixed scale on the card for the fractional amount of the total emittance radiated into a finite spectral band from zero up to some arbitrary wavelength. Marked as $\frac{W_{0-\lambda}}{W_{0-\infty}}$(our $\mathfrak{F}_{0\to\lambda}$), it was expressed as a percentage and contained the range from 0.0001% to 99%. Again a gauge mark labelled "MAX" appeared on the scale at 25% and gave the fractional amount when the interval from zero up to the peak wavelength λ_{max} was considered. A new scale ran along the card above the top of the bottom panel. It was for the ratio of the spectral emittance at any given wavelength λ to that at the peak wavelength λ_{max}. The scale was marked as $\frac{W_\lambda}{W_{\lambda_{\text{max}}}}$. Aligned with this scale was a reverse logarithmic scale for the wavelength identical to that just described. Such an identical scale would not have been needed if the slide chart came with an adjustable cursor.

Instead reliance on direct alignment of each wavelength scale with these two scales was needed in order to be correctly read.

The bottom panel on the front of the rule was quite wide. Running through the middle of this panel on the slide between the two identical wavelength scales above and below it was a black curve representing the transmission spectrum for electromagnetic radiation in the atmosphere. It gave the fraction of incident radiation transmitted (the fraction being expressed as a dimensionless transmission coefficient between zero and one) through the atmosphere as a function of wavelength over a distance of one nautical mile horizontally to the surface of the Earth at approximately 80% relative humidity for an air temperature of approximately 80°F. Finally, the curve in red found in the visible wavelength portion of the spectrum was a relative luminosity curve for the eye and was identical to that found on the GEN-15. On the reverse side of the rule, at the top and to the left of the temperature panel was a very narrow panel corresponding to the wavenumber $\tilde{\nu}$ in units of per centimeter [cm^{-1}]. It converted the peak wavelength found in the spectral emittance curve λ_{max} into wavenumbers $\tilde{\nu}_{\lambda_{\text{max}}} = 1/\lambda_{\text{max}}$.

The remaining two panels found on the reverse side were new additions to the GEN-15B rule. In place of the lower panel found on the GEN-15 for the total emittance in units of kilowatt-hours per square feet per hour and kilocalaries per square feet per hour is a panel with two scales relating to actinometric quantities. The scale along the top of this panel was a reverse logarithmic scale for the total photon emittance M_q in units of photons per square centimeter per second [photon cm^{-2} s^{-1}]. The bottom scale appearing in this panel gives the energy, in electron volts (eV), a single photon has at a wavelength corresponding to the peak wavelength λ_{max}. The final long narrow panel appearing on the reverse side of the slide chart at its base was used to find the irradiance normal to an area of one square centimeter at a certain distance from a blackbody source where losses in the intervening atmosphere were assumed to be negligible. The irradiance was measured in units of watts per square centimeter [W cm^{-2}]. The scale running along the top of the panel was a reverse logarithmic scale for the irradiance at close range. Aligned just above this scale on the card was the range in centimeters from 90 cm to 1.26×10^5 cm. The scale running along the bottom of the panel was also a reverse logarithmic scale for the irradiance at ranges beyond a kilometer. Aligned just below the scale on the card was the range in nautical miles from 0.38 to 1000 nautical miles. For convenience a gauge mark at one kilometer appeared on the nautical mile range scale.

Once again some useful information to aid the user appears on the GEN-15B. On the reverse side in the top left corner the three equations for Planck's law, Wien's displacement law, and the Stefan–Boltzmann law are again given. Below these three laws the Stefan–Boltzmann constant is once more given in seven different system of units. Starting in the top right corner and listed vertically downwards are sixteen different radiometric symbols and physical constants. The values for the seven physical constants that appear have also been updated compared to those found on the GEN-15 and suggest the scales found on the later Radiation Calculator should be more accurate compared to those found on Canada's initial rule. Though not mentioned, the updated values used for all physical constants correspond with those given in 1953 by Jesse William Monroe DuMond and E. Richard Cohen.[53] Two different panel variations found at the front top left-hand end of the GEN-15B appeared. The first of these came "courtesy of Light Military Electronics Equipment Department, General Electric, Utica, NY" the second was "compiled by Optics and Color Engineering Component, General Engineering Laboratory, General Electric, Schenectady, NY". As late as 1964 the rule was being advertised for sale for a cost of $1.50.[54,55]

A third and final iteration of the slide chart for General Electric first appeared sometime in 1965.[56] Designated the GEN-15C it was identical in form, in both construction and in the number of scales used, to the GEN-15B. Finding two different designations for what were essentially identical slide charts may initially seem a little odd. However the most likely reason for this discrete change lay in an unreported error in one of the new scales found on the GEN-15B. When Canada introduced his rule in 1948 he characterised it as being suitable for order-of-magnitude calculations. It turned out his rule was usually a lot better than this, often giving estimates with an error that was less than 1%. The scales on the GEN-15B had been updated, having taken into account the 1952 least-squares adjusted values for the physical constants as given by DuMond and Cohen. Accurate readings for many of the quantities found on the rule however not only required a certain degree of accuracy in the scales themselves but also in the positioning of the gauge marks on the card from where the reading was made. The two gauge marks used on the actinometric panel of the GEN-15B were incorrectly positioned. Accordingly, estimates for both the total photon emittance $M_q(T)$ and the spectral photon emittance at the peak wavelength $M_{q,\lambda_{\text{max}}}$ were in error anywhere upwards of 15%. While this may not seem large, its significance can be better appreciated by highlighting some of its potential consequences. For example, the "generous" nature of the GEN-15B would yield infrared sensors that performed worse than expected and caused some companies at the time significant financial grief and embarrassment. The release of the GEN-15C finally corrected the ill-positioned gauge marks found on the two actinometric scales but not before it had earned the nickname "Generous Electric". The error does not seem to have ever been formally acknowledged, at least in the literature, though engineers working in the infrared at the time were strongly warned to always check and be sure the Radiation Calculator they were using was not the GEN-15B.

Like the GEN-15B, the GEN-15C is known to have come in at least three different front panel variations. The first was the same as that found for one of the GEN-15B's, it coming "courtesy of Light Military Electronic Equipment Department, General Electric, Utica, NY". The second came "courtesy of Aerospace Electronics Department, General Electric, Utica, NY", while for the third there simply appeared "General Electric". A photograph of the front and back faces of the GEN-15C are shown in Fig. 5.

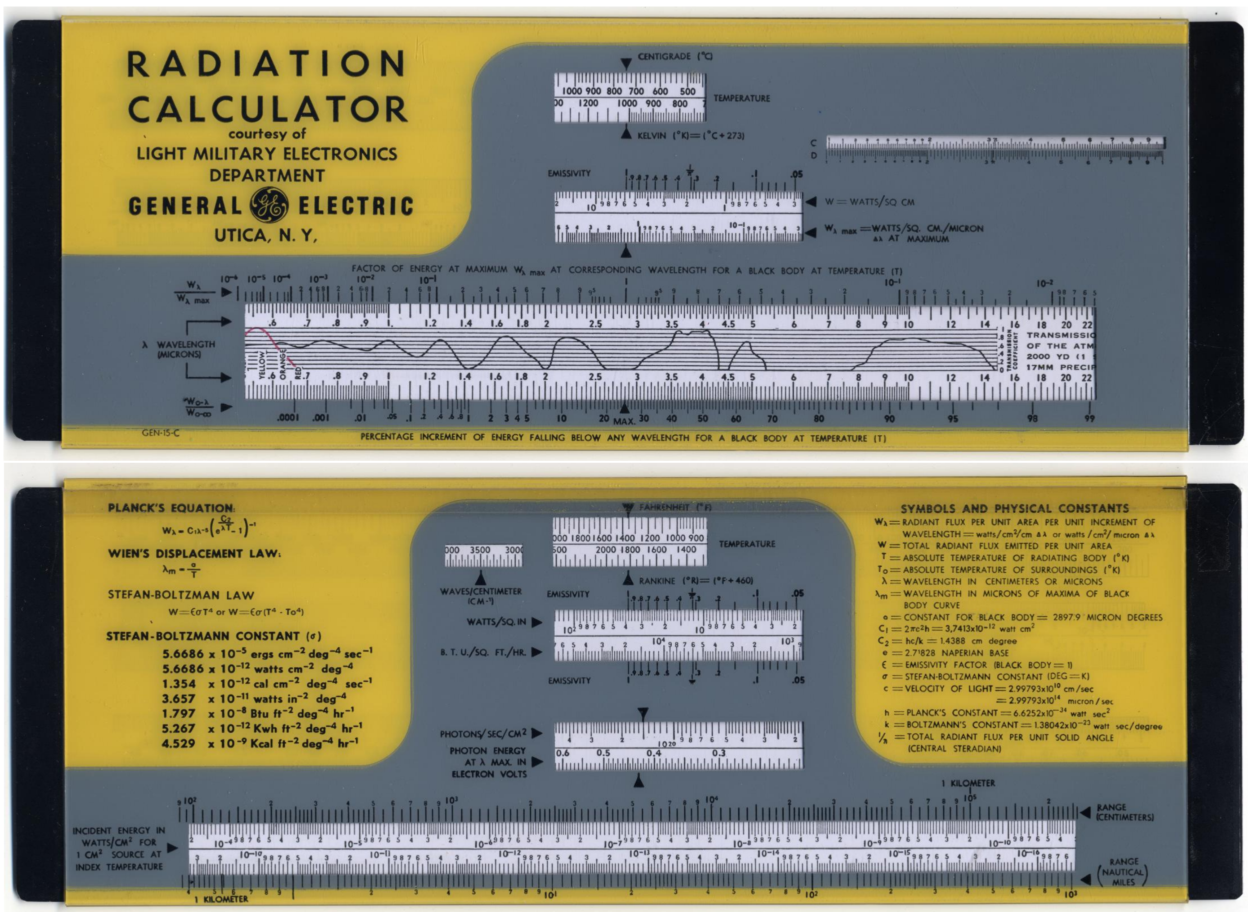

Figure 5. Front (top) and back (bottom) faces of the General Electric slide chart GEN-15C.

The radiation slide rule designed by Canada was by far the best known example of its kind in the US. By the early 1960s its adoption and use in the US was widespread. In 1962, John N. Howard noted the slide chart from General Electric was "...usually found in every infrared man's briefcase".[46] Some years later, Richard D. Hudson in his text *Infrared System Engineering* went further. He observed Canada's rule was "...virtually a badge of the infrared fraternity", and despite Canada himself characterizing his rule as suitable for only order-of-magnitude calculations, suggested around 90 per cent of all thermal radiation calculations performed in the US in the decade leading up to 1969 were made with one of the various slide charts from General Electric.[47] Descriptions of the GEN-15C slide chart were also to be found in a number of widely read infrared texts and manuals of the day suggesting their authors felt the need for those working in the field to at least be acquainted with its existence.[47,56–59] The importance of the rule is also reflected in the number of citations made to it in the literature after 1948. While it is true many who made use of the slide chart may not have referred to it explicitly, in the decades following its introduction up until the mid-1970s it was widely cited. As a relatively inexpensive device that at best was only capable of producing estimates accurate to half a per cent, its continued longevity into an age of increasingly affordable computing power was all the more remarkable. In the mid-1980s one finds Lewis J. Pinson in his text *Electro-Optics* strongly urging anyone considering to do serious work in the infrared to acquire the Radiation Calculator from General Electric.[60] Two decades later the GEN-15C Radiation Calculator was still being referred to. It was in a paper outlining a sample return mission to the two Martian moons Phobos and Deimos using a proposal based on contemporary solar sails.[61] More recently still, in the text *The Art of Radiometry* written in 2010 by James M. Palmer and Barbara G. Grant,[62] an image of the front face of the GEN-15C is given and is described as the "venerable" Radiation Calculator from General Electric, presumably for the benefit of a younger audience unfamiliar with calculational aids such as slide rules. And it was not just engineers working in the infrared who found the Radiation Calculator from General Electric useful. As an example one finds the many virtues of the rule being extolled to, of all people, advanced photographic technicians.[63]

The success of Canada's radiation slide rule can be attributed, at least in part, to its ease of use, ready availability, and its ability to produce results accurate enough for their intended purpose. In a single setting seven radiometric and actinometric quantities could be directly read from the Radiation Calculator and all accomplished from a highly portable, relatively inexpensive device. Ultimately, Canada's rule spawned a number of successor slide charts after it ceased to be available from General Electric. Each was almost identical in design and form to Canada's later Radiation Calculator and extended the legacy of his rule well into the early 1990s. Made of finished cardboard and often given away as a promotional item these successor *Infrared Radiation Calculator* slide charts appeared in the 1970s and 1980s from firms and organizations such as Sensors, Inc.,[64,65] the Infrared Information Analysis Center (IRIA),[66] and EG&G Judson.[67]

3.3 The A. G. Thornton F5100 radiation slide rule made for the Admiralty Research Laboratory

During the course of his work on thermal detection problems, having found existing tables based on Planck's radiation law neither sufficiently comprehensive nor convenient for frequent use, the Polish émigré Mieczysław Wiktor Makowski,[§] who in early 1945 was working at the Admiralty Research Laboratory in Teddington, England, set about to develop a series of nomograms. Intended as a labour saving device they allowed Makowski to quickly approximate many of the most frequently encountered quantities in thermal radiation. At around the same time of their completion a simple eight inch slide rule made by Professor Czerny of Frankfurt University was received at the Admiralty. At eight inches long its length suggests the rule received was one of Czerny's hand-built prototypes rather than the longer 10048 production model later made by ARISTO. No doubt a fortuitous arrival, to Makowski the advantages of such a simple device were immediately apparent. At once the decision was taken to extend the computations he and his co-worker, the Assistant Experimental Officer L. A. J. Verra, had already made with their intention being the development of a slide rule far superior to the one they had just received. By October 1945, development of the rule had progressed to the point where discussions between one of England's leading slide rule manufacturers, A. G. Thornton Limited of Manchester, and Makowski on behalf of the Admiralty had commenced.[68] The nature of the talks sought to understand what the most suitable form the measurements for the scales ought to take for manufacturing purposes. The issue of copyright was also raised. Here the Admiralty felt copyright should be retained by them over any future produced slide rule in view of the amount of work involved in its preparation.

The first description of Makowski's radiation slide rule appeared in a technical report he wrote for the Admiralty in September 1947.[69] Considerable attention is paid to explaining how the scales were calculated. He notes approximately 1000 man hours of computing time were involved in the preparation of the scales. In finding the positions of major tick marks for each scale these were calculated using one of the equations associated with the quantity developed in the report. Fractional marks between major tick marks were then found via interpolation using Bessel's interpolation formula up to fourth and sometime sixth differences. The majority of the calculations were made using two Brunsviga calculating

[§] Very little is known about MIECZYSŁAW WIKTOR MAKOWSKI. He was born on 5 January 1900 in St. Petersburg, Russia to Polish parents. At the time his paper on the radiation slide rule was published in December 1949 he was working at the Polish University College in London after having spent a number of years working for the Admiralty Research Laboratory in Teddington, England. He is known to have held a Diplom-Ingenieur, the traditional engineering degree from Germany, and was an Associate Member of the Institution of Electrical Engineers in the UK. Through naturalisation he had become a citizen of the United Kingdom and its Colonies in June 1949. In the mid-1950s Makowski is found working at the British Electrical and Allied Industries Research Association, or ERA as it was more generally known (ERA was short for "Electrical Research Association"). It is not known when he died.

machines.¶ Calculations for some of the scales were then checked using one of Britain's very early electronic computers located at the recently established Mathematics Division of the National Physical Laboratory, a close neighbour of the Admiralty's at Teddington. In his report Makowski mentions three prototypes had been constructed, each being capable of calculating a great many quantities relating to thermal radiation. Experience in working with these prototypes however convinced Makowski a reduction in the total number of scales on the final commercially available rule was needed. Though not mentioned its likely the absence of any scale relating to radiance on the final rule, even though it was one of the scales found on Czerny's rule, was removed from the prototype. Burdened by their inclusion Makowski saw they could be readily done away with since for a blackbody each can be obtained from the emittance on dividing by π. Doing so therefore avoids any unnecessary multiplicity of scales. The technical report continued with a brief description of the slide rule, its scales, and an analysis of its accuracy. A number of examples typical of those encountered in practice were also included so that, as Makowski wrote, the report may serve as an instruction booklet for any future produced slide rule.

The first commercially available rule based on Makowski's design was made for the Admiralty by A. G. Thornton Limited and was designated the F5100 Radiation Slide Rule. The base of the rule was made from Honduras mahogany. White celluloid veneers containing the scales and brief instructions were then pinned and glued onto the wooden base. Its impending availability was reported in ONR's *European Scientific Notes*, in the second August issue of 1948.[70] At the time cost was not known but was expected to be rather expensive and the initial production run was to be limited to 50 rules only, a quantity the Admiralty thought sufficient to meet the needs of other parties engaged in thermal radiation work. The Admiralty were however willing to supply photo-printed sheets of the scales on stabilized card weight stock to any interested institution for about $2 per set and suggests the Admiralty were not expecting much of a demand for their rule. How mistaken they would turn out to be. In 1949 a paper based on a slightly modified version of Makowski's technical report was published in the literature. It appeared in the December issue of *The Review of Scientific Instruments*[71] and was published at a time when the rule was first thought to be available in England. Makowski does not seem to have been aware of Canada's rule as he makes no reference to it. Like Canada's rule before it, references to Makowski's rule in the literature quickly followed.[72] While the rule was at least known in the US, being reviewed in the US publication *Mathematical Tables and Other Aids to Computation* in 1950[73] and cited by US-based authors some years later,[52,74,75] it seems it could only be obtained directly from England until the early 1960s after which time it became available through a number of US-based distributors.[56,76–78] At a cost of around $150 in 1962 it was by no means cheap and meant the rule was typically purchased for shared use amongst the employees of a company or organization involved in infrared work rather than by an individual. A reprint of Makowski's 1949 paper bounded between a lime green cardboard cover served as the instruction manual for the rule. In time, the rule would become known simply as "the Admiralty rule".

Makowski's design for the Admiralty rule was one of the most elaborate of all the radiation slide rules to be made. A total of 23 scales for 14 different quantities can be found on the rule. Like Canada before him, Makowski used Birge's 1941 values for the physical constants in the calculations made for the scales of his rule and is quite explicit in this. Not only does Makowski refer to Birge's work in his paper of 1949, he repeats it again in print on the back of the rule as part of the brief instructions given. Here, at the end of a list of numerical values for the constants used, one finds a reference to Birge but unfortunately the year of publication is incorrectly given as 1944 rather than 1941. Each scale on the Admiralty rule was conveniently lettered from a through to s and we will use these letters when referring to them here. On the front of the stock seven scales appear, four at the bottom (labelled a, b, c, d) and three at the top (labelled e, f, g). Twelve scales are found on the slide which is reversible. Since in construction the rule was of the closed-frame type it meant the slide had to be first removed in order to be reversed. The side labelled ENERGY corresponding to radiometric quantities contained six scales labelled h through to m while a further six scales labelled n through to s on the reverse side of the slide labelled PHOTONS corresponding to actinometric quantities can be found. The last three scales are located on the back of the stock running across the top of the rule but are not labelled. Relevant formulae, table of values for six physical constants, and a set of brief instructions for operation of the rule also appear on the reverse side of the rule. Accommodating all these scales meant the rule was rather large, measuring 45 cm long by 9 cm wide by 14 mm thick. The slide was a further 4 cm longer at 49 cm. Even a sizeable cursor made of perspex came with the rule, it measuring 4.5 cm across by 9.5 cm high. Figure 6 shows a photograph of the right hand end of the front face of the A. G. Thornton F5100 radiation slide rule. Here the ENERGY side of the slide is shown.

¶*Brunsviga* was the brand name of a popular series of mechanical calculating machines produced by the German manufacturer Grimme, Natalis & Co. from the 1890s onwards. By appropriately settings the rotors to some initial state and cranking a handle, the machine performed multiplication by repeated addition.

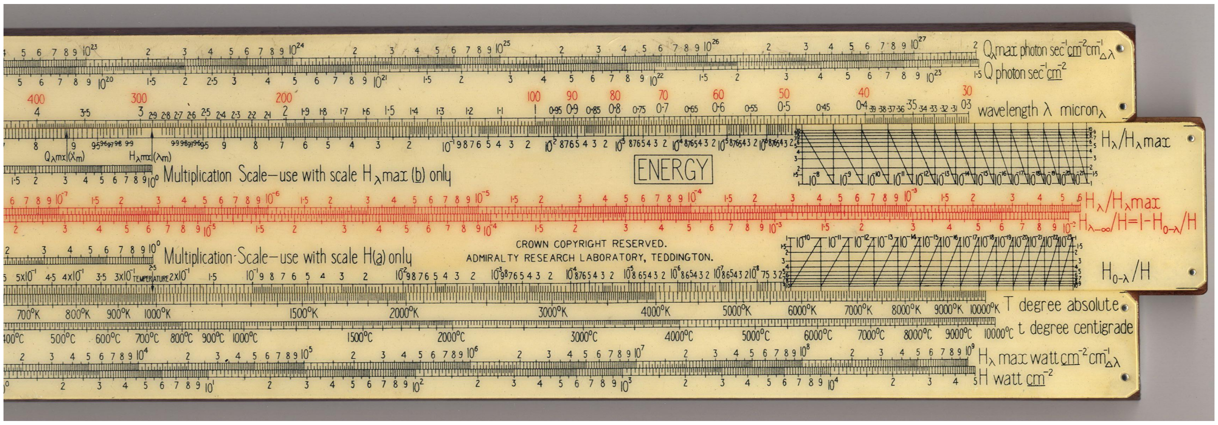

Figure 6. Front right hand end of the A. G. Thornton F5100 radiation slide rule made for the Admiralty Research Laboratory. The ENERGY side of the slide is shown.

The scales together with their associated label found on the Admiralty rule are given below. Note Makowski uses the symbol H for those quantities relating to emittance (possibly for irradiance which was the symbol used for this radiometric quantity at the time) and Q for those relating to photon emittance.

- a Total emittance, H [W cm^{-2}]
- b Spectral emittance at the peak wavelength, $H_{\lambda_{\rm max}}$ [W cm^{-2} cm^{-1}]
- c Temperature in degrees Celsius, t [°C]
- d Temperature in Kelvin, T [K] (100 − 10 000 K)
- e Wavelength, λ [μm] (Black: 0.3–30 μm, Red: 30–3000 μm)
- f Total photon emittance, Q [photon s^{-1} cm^{-2}]
- g Spectral photon emittance at the peak wavelength, $Q_{\lambda_{\rm max}}$ [photon s^{-1} cm^{-2} cm^{-1}]
- h Spectral fractional amount in energetic units, $H_\lambda/H_{\lambda_{\rm max}}$ (for $\lambda = 0.3$ to 30 μm)
- i Spectral fractional amount in energetic units, $H_\lambda/H_{\lambda_{\rm max}}$ (for $\lambda = 30$ to 3000 μm)
- j Multiplication scale for use with scale b only
- k Multiplication scale for use with scale a only
- l Integrated fractional amount in energetic units, $H_{\lambda-\infty}/H = 1 - H_{0-\lambda}/H$ (for $\lambda = 30$ to 3000 μm)
- m Integrated fractional amount in energetic units, $H_{0-\lambda}/H$ (for $\lambda = 0.3$ to 30 μm)
- n Spectral fractional amount in photonic units, $Q_\lambda/Q_{\lambda_{\rm max}}$ (for $\lambda = 0.3$ to 30 μm)
- o Spectral fractional amount in photonic units, $Q_\lambda/Q_{\lambda_{\rm max}}$ (for $\lambda = 30$ to 3000 μm)
- p Multiplication scale for use with scale g only
- q Multiplication scale for use with scale f only
- r Integrated fractional amount in photonic units, $Q_{\lambda-\infty}/Q = 1 - Q_{0-\lambda}/Q$ (for $\lambda = 30$ to 3000 μm)
- s Integrated fractional amount in photonic units, $Q_{0-\lambda}/Q$ (for $\lambda = 0.3$ to 30 μm)

In its design the temperature scale in Kelvin (scale d) was taken as the basis for the rule. As a logarithmic scale it meant the scales a, b, c, f and g were also logarithmic while the eight scales relating to integrated fractional amounts (scales h, i, l, m, n, o, r, s) were not. Along scales h and n four gauge marks for the peak wavelengths in the spectral curves in energetic and photonic units for two different spectral representations are found. The usual peak wavelengths in the linear wavelength representation in energetic units (labelled $H_{\lambda\,{\rm mx}}(\lambda_{\rm m})$) and photonic units (labelled $Q_{\lambda\,{\rm mx}}(\lambda'_{\rm m})$) are given as are those corresponding to the peak wavelengths when a spectral representation in the linear wavenumber representation is considered. These are labelled $H_{\nu\,{\rm mx}}(\lambda''_{\rm m})$ for energetic units and $Q_{\nu\,{\rm mx}}(\lambda'''_{\rm m})$ for photonic units and were very unusual gauge marks not found on any other radiation slide rule. Gauge marks labelled "TEMPERATURE" on scale m on the

ENERGY side and scale s on the PHOTONS side of the slide allowed these gauge marks to be aligned to the desired temperature on the stock. In operation, once the temperature on either scale c or d was selected and the cursor moved over it so the two coincided, from a single setting five quantities could be immediately read from the rule. These were: (i) the peak wavelength using the gauge mark located on scale h or n, (ii) total emittance using scale a, (iii) spectral emittance at the spectral peak wavelength using scale b, (iv) total photon emittance using scale f, and (v) spectral photon emittance at the spectral peak wavelength using scale g. If, on the other hand after the desired temperature had been set, moving the cursor to a given wavelength on scale e allowed the various fractional quantities located on either side of the slide to be determined. The range on the wavelength scale was divided into two parts. The first in black corresponded to wavelengths from $0.3-30$ μm while the second in red corresponded to wavelengths from $30-3000$ μm. Color matching between either the black and red wavelength scales on the stock with identically colored scales for the fractional quantities on the slide was required when reading these scales. On the back of the rule running across the top of the stock three unlabelled scales relating to the wavelength and wavenumber of an electromagnetic wave and the energy the corresponding photon would have in electron volts are given. As these three scales were read without the aid of a cursor, for convenience, the wavenumber scale was given twice.

It was possible to use the Admiralty rule for calculations involving temperatures outside the limits of 100 to 10 000 K using a simple extension process. For a temperature T outside the range, a temperature T_s is selected so that it falls within the temperature range of the rule such that $T_s = T \times 10^n$. Here n is a conveniently chosen integer. Values for the various quantities are then found from the slide rule using the temperature value T_s. Values appropriate to the original temperature T can then be found using the following transformation rules.

$$H_\lambda = (H_\lambda)_s \times 10^{-5n} \qquad Q_\lambda = (Q_\lambda)_s \times 10^{-4n}$$
$$H_{\lambda_1-\lambda_2} = (H_{\lambda_1-\lambda_2})_s \times 10^{-4n} \qquad Q_{\lambda_1-\lambda_2} = (Q_{\lambda_1-\lambda_2})_s \times 10^{-3n}$$

As a reminder, these transformation rules were listed on the back of the slide rule.

The accuracy of the rule varied according to the region of the scale in use. In general, calculations made at longer wavelengths (those made using the red wavelength scale) were accurate to within 1%. In the region of the respective spectral maxima, the relative error reduced to no more than a few tenths of one per cent. For shorter wavelengths (those made using the black wavelength scale) the relative error in the rule gradually increased to about 5%. To help prevent a reduction in the accuracy at these shorter wavelengths a diagonal scale at one end of the slide for scales h, m, m and s (the ratio scales) were introduced and meant values for these ratios at incredibly small values could still be calculated to reasonable accuracy.

The four multiplication scales (scales j, k, p, and q) found on the rule at first appear a little mysterious. In both his technical report of 1947 and his paper of 1949 Makowski seems a little cryptic in his remarks describing how these scales may be useful to the user. Nor is any clue to their use to be found in the examples he provides as none make use of these scales. He writes, "[T]he multiplication scales are so arranged that it is unnecessary to move the slide, when set at a given temperature, in order to carry out the operation of multiplication".[71] While this is true why one would want to do such a thing is not revealed. Of course, to the infrared worker it was immediately apparent these scales could be used to find the total emittance and the spectral emittance at the peak wavelength for greybodies. Interpreting the multiplication factor, which is a number between zero and one, as a value for the object's emissivity meant the emittance for a greybody could be found in both energetic and photonic units.

An intriguing oversight on the part of Makowski is to be found in a short erratum that appeared four months after his paper of 1949 was published.[79] In it he notes a footnote should have been added to the paper stating the work was carried out at the Admiralty Research Laboratory in Teddington, England. We find it strange he should fail to mention this. After all, it was the organization for whom the rule was designed and is where all the work for it took place.

From its inception in the late 1940s up until the early 1960s the rule for the Admiralty was made by A. G. Thornton Limited from Wythenshawe, Manchester. Starting in 1965 the rule continued to be produced by the firm Blundell Harling Limited, another well-known slide rule manufacturer from Weymouth in England. A notable feature of the Blundell Harling Limited produced rule was its improved construction, it now being made completely of plastic. The scales on the rule however do not seem to have accounted for the latest values of the fundamental constants known at the time as the reverse side of the updated rule still lists the 1941 values from Birge. Nor is the error in the year of publication of 1944 for

Birge's values corrected. When the rule finally ceased to be commercially available is not known but it is more than likely it was produced by Blundell Harling Limited right up until the end of the slide rule era in the late 1970s.

Everything about the Admiralty rule was imposing. Grand in scale and ambitious in design it was by far the most elaborate rule of its day. Not until it was finally displaced by a similar though throughly updated rule released in the US in 1970 for Electro Optical Industries, Inc., and shortly thereafter by affordable, hand-held programmable calculators, for a time the Admiralty rule was unsurpassed and made it the tool of choice for very accurate thermal radiation work. A prohibitively high cost however limited its general availability and meant compared to its far cheaper rivals, in particular the Radiation Calculator from General Electric, was not as widely used as it could have been. Short descriptions of the rule are to be found in some of the more comprehensive infrared texts of the day[47,56] and whenever the Admiralty rule was referred to by authors in the literature it was always with the highest praise. Even today Makowski's rule continues to draw attention from slide rule enthusiasts partly due to its size and partly due to the elaborate nature of its scales.[80,81] Reflecting some 40 years after his seminal paper with William Shockley on the thermodynamic energy conversion efficiency limit of a solar cell,[82] Professor Hans Joachim Queisser recalls how in 1960, stuck in a small, cramped, rented office which was a converted old apricot barn in Mountain View, California, he calculated the solar efficiency curve (the curve is now known as the *Shockley–Queisser limit*) by hand using nothing more than Shockley's trusty Admiralty rule.[83] At the time, in the absence of any computer, it was an exceedingly tedious calculation requiring Eq. (13) to be evaluated many times and each time to a reasonable level of accuracy. Queisser, who incidentally was the successor of Professor Marianus Czerny at Frankfurt University, recalls[84] to achieve this level of accuracy required space – physical space that is on the grandest of all the radiation slide rules!

3.4 The Autonetics Photon Calculator

Compared to other radiation slide rules the Autonetics Photon Calculator is sui generis. Circular in form it was designed to calculate essentially one actinometric quantity. This was $M_{q,0\to\lambda}$, the incremental photon emittance for a blackbody within a finite spectral band. While many other radiation slide rules readily provided estimates for the corresponding normalized fractional amount, no other rule was capable of calculating this quantity directly as an absolute amount. Designing a rule to accurately estimate an absolute amount whose value could span up to 45 orders of magnitude was no simple task and to achieve this the circular format of the slide rule was essential. Writing $M_{q,0\to\lambda}$ in scientific notation it takes the form $a \times 10^b$. Here the mantissa a is such that $1 \leq a < 10$ while the exponent b is an integer. Doing so results in the problem becoming more manageable as finding the mantissa and the exponent separately is far simpler than having to find a single absolute amount which spans possibly many tens of orders of magnitude. The Autonetics Photon Calculator managed to do this in a unique and rather clever way.

Autonetics started out in 1945 as a small unit in the Technical Research Laboratory located in the engineering department of the Los Angeles division of North American Aviation, Inc. As a result of it winning a US Army Air Corps contract to develop guided missiles the unit rapidly expanded and led to the establishment of Autonetics as a separate division of North American Aviation in 1955. It was first located in Downey, California, before moving to Anaheim, California, in 1959. In September of 1967 Autonetics became part of North American Rockwell Corporation. Six years later saw its name change to Rockwell International. A number of divisions were included within Autonetics itself. One of these was the Electro Sensor Systems Division. The division built multi-function radar systems, armament control computers, and sensor equipment that made use of photon detectors, among other things, and it was the Electro-Optical Laboratory within the Electro Sensor Systems Division of Autonetics responsible for the development of its photon calculator. Almost all the information we have about the Autonetics Photon Calculator comes from an article that appeared as a substitute to Frank Cooke's regular, and what turned out to be long running, column "Optical Activities in Industry" in the journal *Applied Optics*. It was written by Howard J. Eckweiler[‖] and appeared in the July 1968 issue of *Applied Optics*.[85] As the only author one presumes he was responsible for the initial design and subsequent development of the calculator. Although Eckweiler

[‖]Dr HOWARD JESSE ECKWEILER was born in New York City on 11 July 1906. In 1928 he obtained a Bachelor of Science degree from New York University before joining the Electrical Testing Laboratories in New York City as an assistant to the technical director. In 1935 he joined the mathematics department at New York University. After completing a Master of Science in 1937 followed by a PhD at the Courant Institute of Mathematical Sciences in 1942 he returned to industry, joining Kollsman Instrument Corporation in Elmhurst, New York, as Chief of their Optical Section. In 1958 he left Kollsman Instrument Corporation to found Lyle Co. Some time in the early 1960s he moved to Autonetics, a Division of North American Aviation, Inc. in Anaheim, California, where he assumed the role of a Senior Staff Scientist. He died on 20 December 1996 in Orange County, California, aged 90.

credits Richard Ramsey, TRW, and A. L. Dunklee for providing contributions to the formulation and development of the rule when at Autonetics, it seems the bulk of the work and responsibility rested with him. The copyright on the reverse side of the calculator is dated 1967, North American Aviation, Inc. Interestingly, Eckweiler's paper was received on 15 September 1967, the same month North American Aviation, Inc. became part of North American Rockwell Corporation, and it is the latter name Eckweiler uses as his institutional address in his paper. All this suggests the design and development of the Photon Calculator was probably completed in late 1966 and manufactured some time in early 1967 just before Rockwell-Standard Corporation acquired and merged with North American Aviation, Inc. to form North American Rockwell Corporation. Who the calculator was made by is not known.

The Autonetics Photon Calculator was a circular slide rule made of plastic and finished cardboard. It consisted of a smaller 6¾ inch diameter inner disc made of plastic that moved relative to a fixed outer 8 inch diameter disc made of finished cardboard. A movable radial cursor concentric with the two discs completed the rule. Four color-coded scales appeared on the front of the rule while brief operating instructions and six examples using the calculator took up the entire available space on the back of the rule. On the rim of the larger fixed outer disc were located two scales. The outer most scale was light green and was for the mantissa of $M_{q,0\to\lambda}$. It was labelled $Q^*_{0-\lambda}$ and was in units of photons per second per square centimeter [photons s^{-1} cm^{-2}]. It was a three-cycle ($1 \le Q^*_{0-\lambda} < 10$) closed logarithmic scale. Running around next to this scale on the larger fixed outer disc was a buff colored scale for temperature. It was labelled T and was in Kelvin. It was a single cycle, closed logarithmic scale. As closed scales this meant there was no limit on the values for each of these quantities. One could simply keep going around and around, mentally shifting the position of the decimal place as you went. The outer most scale on the smaller inner disc was green and was for the wavelength λ in micrometers. Again it was a single-cycle, closed logarithmic scale. Printed on this scale marking its start was a large black arrow with the numeral "1" appearing within it. The final white scale on the inner disc was a 33-cycle open logarithmic scale wound in eleven concentric rings for the wavelength–temperature product λT. It was measured in micrometers Kelvin [μm K]. As an open scale its range was limited to: $171.6 \le \lambda T < \infty$ μm K. A photograph of the front face of the calculator is shown in Fig. 7.

In calculating the main scale used on the calculator the integral appearing in $M_{q,0\to\lambda}$ was first converted into an infinite series and its logarithm (since all scales appearing on the calculator are logarithmic) was evaluated using one of the company's computers. The values Eckweiler takes for the fundamental constants are those that had been recently recommended by the National Academy of Sciences National Research Council Committee on Fundamental Constants in 1963.[86] In terms of the accuracy of his calculator Eckweiler says surprisingly little other than remarking he expected values found using it would exceed those of experiment. It was a bold statement backed up with no numerical analysis.

Operating the calculator to find $M_{q,0\to\lambda}$ was a two part process. The first part involved the determination of the mantissa, the second required finding its corresponding exponent. The temperature setting was made by aligning the required value for the temperature on the buff scale with the large black arrow found on the green wavelength scale. The radial cursor was then moved over to the wavelength of interest from where the wavelength–temperature product could be read off the temperature scale. Note since both the wavelength and the temperature scales were single-cycle closed logarithmic scales they functioned in exactly the same manner as the C/D scales used for multiplication on an ordinary slide rule. The preceding step where the cursor is moved to find the wavelength–temperature product was only required if the product could not be performed mentally in one's head. Without moving either disc so the temperature setting remained in place, the radial cursor is then swung around to the corresponding wavelength–temperature product on the inner white scale and $Q^*_{0-\lambda}$ read at the position of the cursor line on the light green scale. With the mantissa found, finding the exponent b was broken up into three steps so that $b = a_1 + a_2 + a_3$. The three different a_i's were integers which appeared in a host of blue circles found at various places on the slide rule and its cursor and were selected by following a set of rules. Values for a_1 were found on a cursor table at the end of the radial cursor. The positive blue circle integer lying between the temperature range shown on the cursor table for the selected temperature setting was chosen. A total of fourteen positive integer values for a_1 from $+12$ to $+25$ can be found here corresponding to temperatures ranging from 1.874 K to 87 000 K. The values for a_2 were located at 120° intervals of arc on each of the eleven concentric rings found on the white scale at the clockwise end for each of the 33 cycles. At the wavelength–temperature setting on the white scale, the first blue circle encountered by moving in a clockwise direction around the concentric ring is the value taken for a_2. There were a total of 33 negative integer values for a_2 ranging from -1 to -33. Finally, the value for a_3 was found as follows. Starting at the wavelength–temperature setting, the first blue circle one meets on either the buff temperature or green wavelength scales moving in a clockwise direction is the value taken for a_3. On the green wavelength scale there

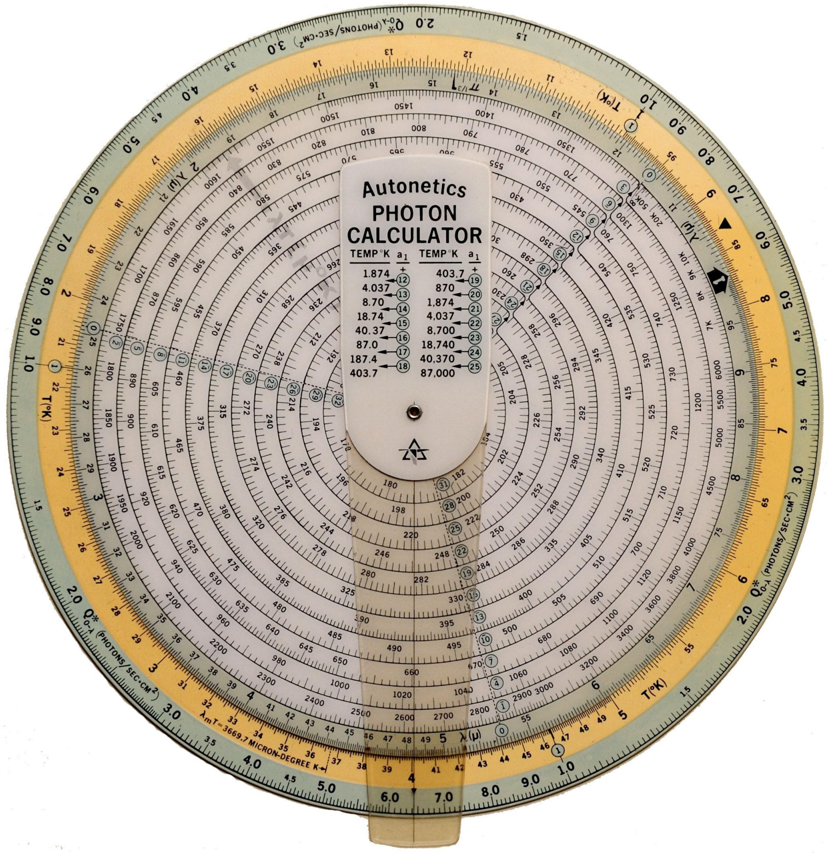

Figure 7. Front face of the Autonetics Photon Calculator made for North American Aviation, Inc.

were three blue circles corresponding to the integer zero while on the buff temperature scale there were three blue circles corresponding to the integer one. Assembling the three pieces by taking their sum gave the final value for the exponent.

In addition to the large black arrow containing the numeral one printed on the green wavelength scale, three other gauge marks appeared on the rule. The first of these was a radial dashed line marked "$\lambda_{\rm m}T = 3669.7$ MICRON-DEGREE K" found on the buff temperature scale. It gave the peak wavelength in the photon spectral curve $\lambda_{q,\max}$. Setting the large black arrow to some appropriate temperature followed by aligning the cursor over this gauge mark the peak wavelength is the value read off the green wavelength scale. The second of the gauge marks labelled $\pi^{1/3}$ was located at 1.4646 on the green wavelength scale. It allowed the corresponding photon radiance for a blackbody to be found from the photon emittance by dividing the latter by a factor of π. To do this on the calculator, with the radial cursor held in place at the value for $Q^*_{0-\lambda}$ found on the light green outer most scale the inner disc was rotated until the $\pi^{1/3}$ gauge mark was brought exactly under the cursor line. With both discs now held fixed the cursor was moved until it aligned with the large black arrow on the green wavelength scale. The reading made from the cursor in this position at the light green scale gave the corresponding value for the mantissa of the photon radiance. The exponent remained unchanged and was equal to the value found previously for the photon emittance. The third gauge mark was a large black triangle located on the buff temperature scale. It was used to find the fractional amount $\mathfrak{F}_{q,0\to\lambda}$ as follows. The large black arrow on the wavelength scale was firstly aligned to the large black triangle on the temperature scale. With the wavelength–temperature product calculated in advance and both discs held fixed the radial cursor was then set to this value on the white scale. With the cursor at this setting the value for $Q^*_{0-\lambda}$ on the light green scale was read. The value for the fractional amount was then equal to $Q^*_{0-\lambda} \times 10^{a_2}$. Here the value for a_2 was found in the same manner as described previously. Lastly, it was also possible to calculate the total photon emittance M_q using the calculator. With the large black arrow on the green wavelength scale set to the appropriate temperature setting on the buff scale, with both discs held fixed, moving the cursor to the "∞" setting on the white wavelength–temperature product scale and reading the value of $Q^*_{0-\infty}$ from the light green scale gave the value for the mantissa corresponding to the total photon emittance. The exponent was then found in a manner identical to that described earlier by finding integer values for the three different a_i's.

The Autonetics Photon Calculator was a specialized rule within an already specialized family of radiation slide rules. While it may not have been able to find many thermal quantities, those it could find were found quickly and efficiently. One obvious advantage to using the Autonetics Photon Calculator lay in its ability to perform many repetitive actinometric calculations in a highly efficient manner and made it particularly suited to someone working on problems relating to photon detectors. Unlike other radiation slide rules, the photon calculator came relatively unencumbered since it was not burdened with any unnecessary scales that were neither needed nor used. It was also very useful for calculating actinometric fluxes at very low temperatures. How widely the Autonetics Photon Calculator was used is difficult to assess. Reference to it in the literature are few and far between. Hudson in his 1969 text *Infrared System Engineering* devotes two very short sentences to it.[47] Importantly, he indicates in a footnote how reproductions of the calculator could be obtained by inquiring through the Electro-Optical Laboratory of Autonetics. As one of us (RBJ) recalls the calculators were difficult to acquire and if it had not been for a fortunate visit to Autonetics and being presented with one, we may never have come by the calculator in the first place. Further references to it are made by Keyes and Quist[87] who suggest some basic calculations related to the performance of photon detectors in the infrared region are best handled with the aid of the Autonetics Photon Calculator while Pinson as late as 1985 recommends the Autonetics Photon Calculator be used whenever one was required to calculate actinometric quantities.[60] When asked recently of the ten different types of radiation slide rules he had, of these which did he use the most and which did he considered to be the best of the bunch, Prof. Wolfe responded by saying that although he may have used Canada's rule the most, it was the photon calculator from Autonetics he found the most powerful.[88]

3.5 The Infrared Slide Rule slide chart made for Infrared Industries Inc.

Very little has been recorded in the formal literature about the Infrared Slide Rule, a slide chart made for Infrared Industries (IRI), Inc. in the early 1960s. It seems to have been used mainly as a promotional item given away by the company. Even so, the slide chart still managed to find its way into the hands of many. It was often given away in bulk to individuals involved in the teaching of infrared-related courses at universities. Gradually these would have been handed out to a considerable number of students over a period of time, further increasing the extent of its distribution. As a smaller, more diminutive slide chart it was clearly overshadowed by its much bigger brother, the Radiation Calculator from General Electric, but we do not think its intention was ever to compete with such a rule. Instead, it was seen as a handy aid to have if nothing more than a quick, order-of-magnitude estimate was required.

The history of the rule is intricately tied to the early history of the company for which the rule was made. IRI was founded in the late 1950s in Waltham, Massachusetts, by E. Douglas Reddan. Initially the company produced infrared detecting elements, instruments, and control systems for military and civilian use. The person responsible for the design and development of the rule is thought to have been Arthur J. Cussen,** an employee who joined IRI in 1958. By the late 1950s Cussen already had a close connection with radiation slide rules. At the end of the Second World War he was with the US Navy, eventually becoming the head of the Infrared Division at the Naval Ordnance Laboratory located at Corona, California. During his time with the Navy he first learned of Makowski's rule made on behalf of the British Admiralty which saw him becoming partly responsible for making the availability of the rule better known in the US.[89] In 1958 he left the Naval Ordnance Laboratory to become the General Manager of the Infrared Standards Laboratory in Riverside, California (though the plant itself was actually located in Pomona).[90] The laboratory had recently become part of the engineering and production division of IRI in Waltham, Massachusetts. In 1960 Reddan, as company president, decided to establish his residence in Santa Barbara, California. In August of that year this led to the selling of additional stock for the construction and equipping of a new factory in nearby Carpinteria (though the future company address would always be

**Born in St. Louis, Missouri on 23 June 1925 ARTHUR JOHN CUSSEN spent his entire professional career working in the optical and infrared regions of the electromagnetic spectrum. After graduating with a Bachelor of Arts degree in 1947 he got his start in infrared working for the government at the Naval Ordnance Laboratory in Corona, California. After becoming the head of the Infrared Division in 1956 he left two years later to head up Infrared Standards Laboratory, a newly formed subsidiary of Infrared Industries, Inc. in Riverside, California. In 1964 he left Infrared Industries, Inc. to establish Electro Optical Industries, Inc. in Santa Barbara, California. Here he served as the founding company president for just over thirty years before stepping down on retirement in 1995. Known to all who knew him simply as "Art", he now lives with his wife Marjorie in Corpus Christi, Texas.

given as Santa Barbara). On completion in 1962, a reorganization of a number of former divisions of IRI took place and saw Cussen along with William E. Standring, Jr.,who was the General Manager of IRI's Photoconductor Division, being appointed vice presidents of the new IRI plant in Santa Barbara.

Very similar in design to Canada's General Electric rule, the IRI rule was also a slide chart rather than a slide rule despite its designation as an "Infrared Slide Rule". At least two different types that differed mainly in their construction are known. The first was made of finished cardboard like the GEN-15 while the second was made of plastic like the GEN-15B and GEN-15C rules. Who the IRI rules were manufactured by is not known, but judging by their appearance, suggest they were made by Perrygraf. Nor is any year marked on the rules making dating them particularly difficult. One source we have comes from a former employee of IRI. Max J. Riedl, who joined IRI at the new company plant in Carpinteria, recalls a cardboard version of the rule was already there when he arrived in early 1962.[91] He also believes the earlier of the two rules was the one made of cardboard. A later cardboard version of the rule also appeared. It is more recent than the plastic version since on the reverse side of each rule the noise equivalent bandwidth, Δf, is defined and its units given. The plastic rule lists the units for Δf as cycles per second (cps) while the cardboard version gives its units as hertz (Hz). In 1960 the SI unit for frequency was changed from cps to Hz and therefore suggests the former plastic rule is older than the latter cardboard rule.

In size the more recent cardboard IRI rule measured 16.2 cm long by 9.9 cm wide and came with a very simple instruction card printed on either side. On the front of the rule were six panels containing fourteen scales for eleven different quantities. Unlike the General Electric rule whose scales placed most of their emphasis on the estimation of radiometric quantities, the scales on the IRI rule were evenly split between radiometric and actinometric quantities. The top panel located in the middle of the rule was a scale for temperature in Kelvin only. Immediately below the temperature panel, the scale running along the top of the second panel gave, at a selected temperature, the total photon emittance in units of photons per second per square centimeter [photons s^{-1} cm^{-2}] while the scale running along the bottom of the panel gave the spectral emittance at the corresponding spectral peak in units of photons per second per square centimeter per unit wavelength interval [photon s^{-1} cm^{-2} μm^{-1}]. The third panel was identical to the preceding panel except it was for radiometric quantities in energetic units. The two longer panels running along the bottom and second from the bottom gave spectral and integrated ratio amounts. The scale above the bottom panel running along the top of the stock was for the spectral fractional amount in photonic units while the scale running below the panel along the bottom of the stock gave the integrated fractional amount in photonic units. On the slide between these two scales was a scale for the wavelength in micrometers though the unit of measure is not given on the rule itself. The scales found on the panel second from the bottom were again identical to those found on the bottom panel but were for the corresponding radiometric quantities. The final panel, located at the top and to the left of the temperature panel, gave as a wavenumber in units of per centimeter the peak value for the spectral curve in the linear wavelength representation. On the reverse side of the rule, running along its base was a temperature nomogram for converting between Fahrenheit, Celsius and Kelvin. The remainder of the reverse side of the rule was devoted to a listing of a number of useful formulae relating to sensor detectivity and a listing of eleven symbols together with their meaning. On the reverse side of the slide approximate transmissions (for wavelengths of transmission greater than 10%) for various optical materials and their refractive index at 3 μm were given. For the cardboard version, the transmission spectrum for radiation in the atmosphere similar to the one given on the General Electric GEN-15B and GEN-15C rules was also given. To read either of these scales the slide needed to be removed from the rule first.

Some time later, though exactly when is not known, IRI had made for it their second type of Infrared Slide Rule. Identical in size and almost similar in design to its cardboard predecessor it was made from plastic. A flexible plastic slide was now housed in a rigid vinyl sleeve. On the front of the plastic rule a grey, rather than the former white, background was punctuated by five clear panels instead of the six found on the cardboard version. No panels were again found on the back of the rule. The second and third panels for the total emittance and the spectral emittance at the spectral peak in both photonic and energetic units found on the cardboard rule were combined into a single panel on the plastic rule. The temperature panel located at the top of the latter rule was also expanded in two ways resulting in two minor variations. In the first variation, which we will call P1, the temperature panel had a scale for the temperature in Celsius running along the top and a scale for Kelvin running along the bottom. In the second variation, which we will call P2, the temperature panel was expanded further to also include a scale for Fahrenheit. The remaining three scales found on the plastic rule were identical with those found on the cardboard rule. On the back of the plastic rule a list of formulae and symbols are once more given but were oriented in portrait rather than landscape form. Other minor variations found between the two plastic rules include the symbols used on the integrated ratio scales for the total emittance and total photon emittance. On rule

P2 one finds W and Q being used for the total emittance and the total photon emittance respectively, and were the same symbols as those used on the cardboard rule, while on rule P1 these two symbols were changed to $W_{0-\infty}$ and $Q_{0-\infty}$. As was the case with many of the other radiation slide rules discussed, older symbols of W for the emittance and Q for photon emittance were used. On the reverse side of rule P1 the incident flux density H is one of the symbols defined while the symbol P for peak is not given while on rule P2 it is the exact opposite, the symbol P being given while H is not. Finally, at the bottom on the reverse side of both plastic rules the Waltham and Santa Barbara addresses for IRI are given. On rule P1 this appears over three lines while it takes up five lines on rule P2. The front faces for the cardboard (left) and one of the plastic (right) IRI rules are shown in Fig. 8.

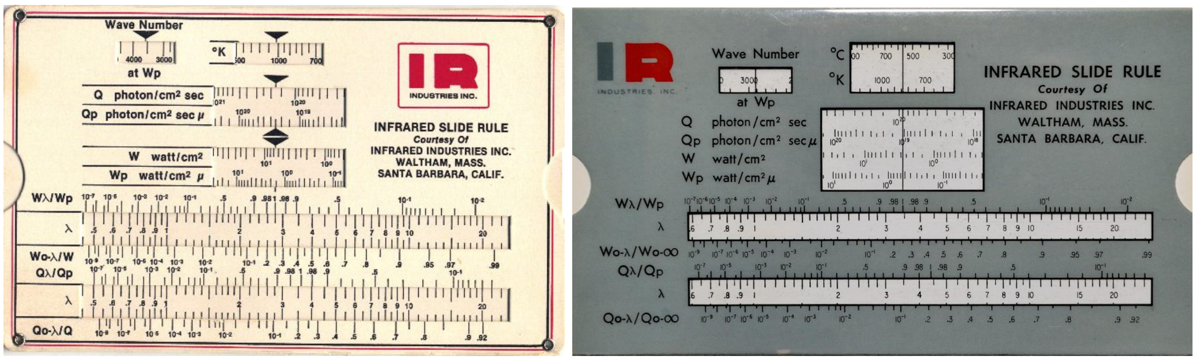

Figure 8. Front face of the Infrared Slide Rule made for Infrared Industries, Inc. To the left is shown the more recent cardboard rule while to the right the plastic version P1 is shown.

Due to their size the IRI rules were without doubt the handiest of all the radiation slide rules made. Given away at trade shows and to students alike, they no doubt found their way into the hands of many potential users. Reserved for very rough order-of-magnitude estimates only explains why one fails to find any reference to these rules in the literature. As the precision with which the scales on the IRI rule could be read was low, it meant any serious work would always be passed over to one of its more advanced and able rivals. It would however not be the last radiation slide rule Cussen would find himself closely associated with. In 1964 he would leave IRI to establish his own company, Electro Optical Industries, Inc.,[92] and it would be from here he would go on to play a pivotal role in the development of the last of the true radiation slide rules, a rule that would finally rival the Admiralty rule.

3.6 The Pickett Model 17 radiation slide rule made for Electro Optical Industries Inc.

The radiation slide rule made for Electro Optical Industries (EOI), Inc. was an improved and updated version of the Admiralty rule designed by Makowski. First available in 1970 it represented the pinnacle of what was possible in a general purpose radiation slide rule. It was also the last of the true slide rules to be made for performing thermal radiation calculations. While several Infrared Radiation Calculators came later these were not slide rules in the strictest sense but slide charts, and since each was almost identical to the Radiation Calculator from General Electric, they simply extended the legacy of Canada's rule without adding anything further to the development of the radiation slide rule. The usefulness of the EOI rule to those working in the infrared would however turn out to be relatively short lived. Through no fault of its own the EOI rule would come to be surpassed in a few short years by the arrival of hand-held programmable electronic calculators.

Established in 1964 EOI had quickly become one of the leaders in the design and manufacturer of infrared and visible sources for testing and calibration equipment in the US. In particular, the company was widely regarded for the quality of the blackbody radiation sources it produced. As a company whose main line of business was intimately connected to blackbody radiation[93] it makes sense they would have more than just a passing interest in developing a radiation slide rule of their own. And if the founder and president of the company in question just happened to be Arthur J. Cussen, this is hardly surprising. By 1970 Cussen's involvement and connection to radiation slide rules was already well established. Having already been responsible for the Infrared Slide Rule that was made for IRI during his time spent working there in the early 1960s, and earlier still in his part responsibly for making the availability of the Admiralty rule better known in the US when he was with the Naval Ordnance Laboratory, it must have always been a desire of his to design an ambitious rule of his own that would one day rival Makowski's Admiralty rule. The rule he had been responsible for designing and producing for IRI was functional perhaps but hardly inspiring and was more toy than high-precision workhorse. The task of starting a new company and taking on the role of its founding president understandably diverted Cussen's attention for a time and meant any immediate ambition he may of had in producing a radiation slide rule of grand design languished for several years. The job of its design and development was eventually taken up in 1968 by the young engineer Raymond J. Chandos[††] who had started working at EOI during the late 1960s, first as a summer intern student, and then full-time after graduating. Cussen's support and backing, who as company president now had the necessary means and resources available to devote to its development, were however vital and without it the rule would have never seen the light of day.

The rule Chandos designed was similar to the Admiralty rule but represented an improvement in several important ways. For a start, the positioning of the scale markings had been updated using the latest known values for the fundamental constants. Secondly, in construction the rule was duplex in form. As a double-sided rule this meant the slide no longer had to be removed to be reversed, one simply flipped the rule over instead. The commercially available rule of 1970 based on Chandos' design was made by Pickett and designated the Model 17. At the time Pickett was one of the larger slide rule makers in the US who was famous for its all yellow "eye-saver" rules made from aluminium. The Model 17 rule, while being made from aluminium, was "traditional" white in color. It measured 30.8 cm long by 5.2 cm wide and as a duplex rule came with a nylon cursor that wrapped around both sides of the rule. Raised end-braces at either end of the rule held the top and bottom stocks in place and prevented the cursor from sliding off. Scales printed in both black and red are found on the rule. In its short life, in honor of its instigator, the rule would be referred to as "Cussen's rule".[59] As was the case with the Admiralty rule, one side of Cussen's rule was used to calculate radiometric quantities while the other calculated actinometric quantities. As with many slide rules sold in the US at the time, it came with the customary leather holster case and an interim manual.[94] A year later the interim manual was replaced with a two color typeset manual.[95] Unlike the Admiralty rule, as a duplex rule it meant there was no available space anywhere on the rule to print a brief set of instructions and made having access to its detailed instruction manual all the more important.

Most of the information we have about Cussen's rule comes from the aforementioned manuals supplied with the rule. The values Chandos used for the fundamental constants when calculating the scales were the revised values given by the National Bureau of Standards in May of 1969.[96] Details on how the calculations for the scales were made are not provided, although of the two references given in the manual but not referred to in the text, the second of these is to the extensive set of tables of blackbody radiation functions compiled by Mark Pivovonsky and Max R. Nagel in 1961.[16] It therefore seems more than likely these were consulted by Chandos during their preparation.

As alluded to earlier, part of the motivation behind the origin of the rule stemmed from a desire by Cussen to have a radiation slide rule of his own design and making. The other was to produce a rule comparable to the Admiralty rule yet more competitively priced. As Chandos recalls,[97] in the late 1960s only Cussen as company president and his brother Robert E. Chandos as vice-president owned copies of the Admiralty rule while everyone else employed at the company worked with the relatively inexpensive Radiation Calculator slide chart from General Electric. The latter were fine for most tasks but for more accurate work the Admiralty rules would need to be "loaned". As Cussen's rule was manufactured by Pickett, a manufacturer of slide rules conveniently located in Santa Barbara not far from EOI, they were able to get the final retail price for the rule down quite a bit. In 1972 it sold for \$50.[89] While it still may have been considerably more expensive than the Radiation Calculator slide chart from General Electric which at the time sold for about \$4,[59] at this price it was still one-third of the cost the Admiralty rule sold for ten years earlier. The rule's availability was widely announced in the literature, typically appearing under the "new products" section of various journals or trade magazines.[89,98–102]

An incredible 34 scales for 13 different quantities are packed onto Cussen's rule, 18 on the front side labelled ENERGY relating to radiometric quantities and a further 16 on the reverse side labelled PHOTONS relating to actinometric quantities. Photographs of either side of the Pickett Model 17 radiation slide rule are shown in Fig. 9. Running vertically down the far left hand end of the rule on either side are the symbols used for the various quantities next to each scale while at the far right hand end the corresponding unit, if any, for each quantity next to the appropriate scale is given. Unlike the Admiralty rule, Chandos unfortunately does not use any labels for the various scales found on the rule. The scales found on Cussen's rule are listed below. Like the Admiralty rule Chandos used the symbol H for those quantities relating to emittance (again

[††]Professor RAYMOND JAMES CHANDOS was born in 1949 and received a Bachelor of Arts degree from the University of California, Irvine. After graduation he worked for a number of years at EOI before leaving in the early 1980s to join REC Corporation, a spin-off of EOI started by the former Vice President and Chief Engineer his brother Robert E. Chandos. He is currently (as of June 2012) a Professor of Electronic Technology in the School of Physical Sciences and Technologies at Irvine Valley College, California.

possibly for irradiance or was it used in homage to its predecessor) and Q for those relating to photon emittance. The order is given as they appear on the rule running from the top down.

Top Stock – ENERGY Side

1. Total emittance, $H_{0-\infty}$ [W cm^{-2}]
2. Spectral emittance at the peak wavelength, H_{λ_m} [W cm^{-2} μm^{-1}]
3. Temperature in Fahrenheit, t_f [°F]
4. Temperature in Celsius, t_c [°C]
5. Temperature in Kelvin, T [K] (100–10 000 K)

Slide – ENERGY Side

6. Spectral fractional amount in energetic units, $H_{\lambda_1}/H_{\lambda_m}$ (black and for $\lambda = 0.3$ to 30 μm)
7. Spectral fractional amount in energetic units, $H_{\lambda_2}/H_{\lambda_m}$ (red and for $\lambda = 30$ to 3000 μm)
8. Multiplier scale for the spectral emittance at the peak wavelength, $M(H_{\lambda_m})$
9. Multiplier scale for the total emittance, $M(H_{0-\infty})$
10. Integrated fractional amount in energetic units, $H_{0-\lambda_1}/H_{0-\infty}$ (black and for $\lambda = 0.3$ to 30 μm)
11. Integrated fractional amount in energetic units, $H_{\lambda_2-\infty}/H_{0-\infty}$ (red and for $\lambda = 30$ to 3000 μm)
12. C scale found on a conventional slide rule

Bottom Stock – ENERGY Side

13. D scale found on a conventional slide rule
14. Wavelength, λ_1 [μm] (Black: 0.3–30 μm)
15. Wavelength, λ_2 [μm] (Red: 30–3000 μm)
16. Wavenumber, ν_1 [cm^{-1}] (Black: 320–40 000 cm^{-1})
17. Wavenumber, ν_2 [cm^{-1}] (Red: 32–400 cm^{-1})
18. RMS Johnson noise potential per root ohm–hertz, $V_n/\sqrt{R\Delta f}$ [V Hz$^{-1/2}$ $\Omega^{-1/2}$]

Top Stock – PHOTONS Side

19. Total photon emittance, $Q_{0-\infty}$ [photons s^{-1} cm^{-2}]
20. Spectral photon emittance at the peak wavelength, Q_{λ_m} [photon s^{-1} cm^{-2} cm^{-1}]
21. Temperature in Fahrenheit, t_f [°F]
22. Temperature in Celsius, t_c [°C]
23. Temperature in Kelvin, T [K] (100–10 000 K)

Slide – PHOTONS Side

24. Spectral fractional amount in photonic units, $Q_{\lambda_1}/Q_{\lambda_m}$ (black and for $\lambda = 0.35$ to 40 μm)
25. Spectral fractional amount in photonic units, $Q_{\lambda_2}/Q_{\lambda_m}$ (red and for $\lambda = 40$ to 4000 μm)
26. Multiplier scale for the spectral photon emittance at the peak wavelength, $M(Q_{\lambda_m})$
27. Multiplier scale for the total photon emittance, $M(Q_{0-\infty})$
28. Integrated fractional amount in photonic units, $Q_{0-\lambda_1}/Q_{0-\infty}$ (black and for $\lambda = 0.35$ to 40 μm)
29. Integrated fractional amount in photonic units, $Q_{\lambda_2-\infty}/Q_{0-\infty}$ (red and for $\lambda = 40$ to 4000 μm)

Bottom Stock – PHOTONS Side

30. Wavelength, λ_1 [μm] (Black: 0.35–40 μm)
31. Wavelength, λ_2 [μm] (Red: 40–4000 μm)
32. Wavenumber, ν_1 [cm^{-1}] (Black: 250–30 000 cm^{-1})
33. Wavenumber, ν_2 [cm^{-1}] (Red: 2.5–300 cm^{-1})
34. Energy of a photon with a wavelength equal to the peak spectral wavelength, E_{λ_m} [eV]

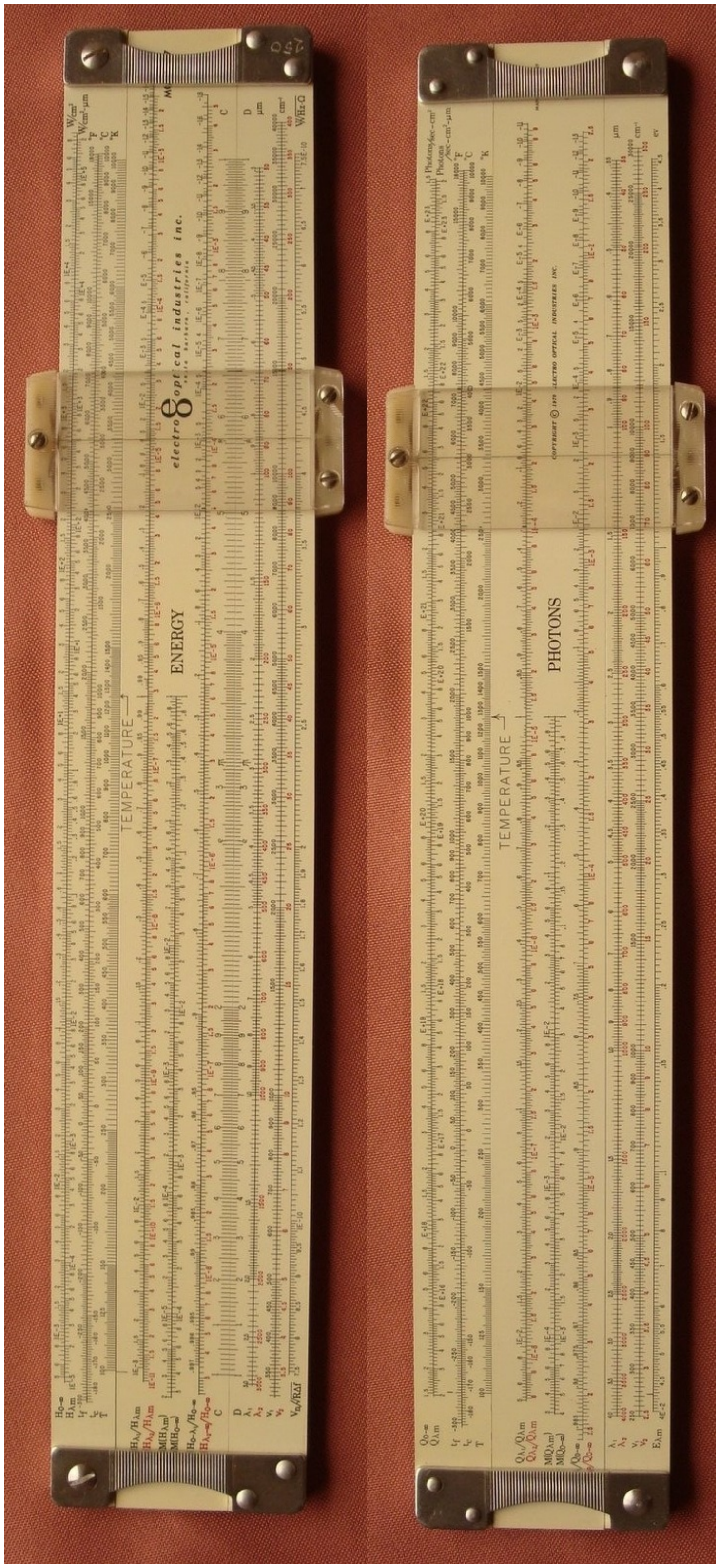

Figure 9. Front and back faces of the Pickett Model 17 radiation slide rule made for Electro Optical Industries, Inc. Photographs courtesy of Robert Adams.

As was the case with the Admiralty rule, the Kelvin temperature scale was taken as the basis for the rule with all other scales on the top and bottom stocks of the rule taking their relative positions from this scale. The range of the wavelength (and wavenumber) scales on each side are not quite identical and there is a very good reason for this. For temperatures between 100 to 10 000 K the peak wavelength in the spectral curve in energetic units $\lambda_{\max}$ lies between 0.28977 and 28.977 μm, almost the range for the wavelength scale λ_1 used on the ENERGY side of the rule, while in photonic units $\lambda_{q,\max}$ lies between 0.36697 and 36.697 μm which is within the range for the wavelength scale λ_1 used on the PHOTONS side of the rule. So for any temperature setting on the rule the corresponding value for the peak wavelength was always to be read from the first of the wavelength scales, λ_1. A single gauge mark labelled "TEMPERATURE" is found on the slide of either side of the rule. In operation the temperature is selected by adjusting the slide to the appropriate setting. After moving the cursor so as to coincide with the selected value for the temperature a total of twelve readings, six on each side, could be made. On the ENERGY side these were: (i) the total emittance using the $H_{0-\infty}$ scale, (ii) spectral emittance at the spectral peak wavelength using the H_λ scale, (iii) the peak wavelength using the λ_1 scale, (iv and v) the corresponding temperatures in both Fahrenheit using scale t_f and Celsius using scale t_c, and (vi) the RMS Johnson noise potential using scale $V_n/\sqrt{R\Delta f}$. On the PHOTONS side the quantities were the same as those given for the first five on the ENERGY side except in photonic units while the sixth reading gave the energy of a photon in electron volts with a wavelength equal to $\lambda_{q,\max}$. For greybodies with emissitivies less than one, the total emittance and the spectral emittance at the peak wavelength could also be found using the two multiplication scales that ran down the middle of the slide on each side. With the temperature set, moving the cursor until its hairline coincides with the value of the emissitivy for the greybody on the multiplier scale, the corresponding emittance for the greybody could be determine from either of the emittance scales running along the top of the stock. If, on the other hand after the temperature had been set, moving the cursor to a given wavelength allowed the various fractional quantities on either side of the slide to be calculated. Importantly, if one was interested in wavelengths found on the black λ_1 scale the black fractional scales on the slide needed to be read. Similarly for the red λ_2 scale and red fractional scales.

Extension in the range of the scale, like with the Admiralty rule, was also possible. Once again, for a temperature T outside of the range a temperature T_s is selected so that it falls within the temperature range of the rule such that $T_s = T \times 10^n$ where n is a conveniently chosen integer. In addition to the extension rules already given, for the scales not found on the Admiralty rule one has

$$E_\lambda = (E_{\lambda_m})_s \times 10^{-n} \qquad \frac{V_n}{\sqrt{R\Delta f}} = \left(\frac{V_n}{\sqrt{R\Delta f}}\right)_s \times 10^{-n/2}$$

Chandos gives no indication to the accuracy of his rule. In one sense it was more accurate than the Admiralty rule since the scales had been prepared using the latest known values for the fundamental constants and therefore put Cussen's rule at a 28 year advantage compared to the Admiralty rule. As the design of the two rules was very similar one would expect their respective accuracies to also to be very similar. However, the greatest difference between the two came in the precision in which the scales on each rule could be read. On the Admiralty rule the Kelvin temperature scale consisted of two logarithmic cycles which was exactly 40 cm long and meant the working modulus chosen in the design of the rule was 20 cm. As all other logarithmic scales on the rule were prepared against this scale it meant their moduli were also 20 cm. Makowski in fact explicitly mentions this and meant, while his rule was rather long, it allowed him to include many intermediate graduation marks in between the major scale tick marks. The Kelvin temperature scale on Cussen's rule on the other hand was only 10 inches long and meant Chandos only had the luxury of a working modulus of 5 inches (about 12.7 cm). This meant it was simply not possible to draw onto the rule as many intermediate graduation marks between major scale tick marks. The precision in which readings could be made using Cussen's rule compared to the Admiralty rule was therefore reduced. The duplex nature of Cussen's rule did however represent an improvement in design as it was possible to read both radiometric and actinometric quantities using a single temperature setting without the need of having to remove and reverse the slide as was the case with the Admiralty rule.

Cussen's rule appeared right at the very end of the slide rule era. A few years later and its realization may never have come to be. The arrival of programmable hand-held electronic calculators at the beginning of the 1970s signaled the beginning of the end of the slide rule's domination. By the mid-1970s programmable calculators had become an almost affordable item, putting them within reach of many engineers. Surprisingly, it is Chandos himself we find leading the charge into the digital age. In 1975 he prepared two short manuals for the Hewlett-Packard 65 and the Texas Instruments

SR-52 programmable calculators that could calculate all quantities found on the Pickett Model 17 rule.[103,104] Not only were these programs much faster, they were accurate to a greater number of significant figures not possible compared to readings made from a slide rule. Here the HP-65 was accurate to five significant figures while seven significant figure accuracy with the SR-52 was possible. But all was not quite lost. In 1979 John N. Howard suggested that even with these fast and accurate programs available it was still useful to have a blackbody radiation slide rule on hand as it gave one a "... better physical feeling for what one was calculating".[105] Wolfe refers to the rule in 1978, together with the GEN-15C, but they found themselves no longer receiving the attention they had once commanded. The amount of space he devotes to discussing an "antiquated" technology the slide rule now represented was small compared to the space he devotes to program listings for the Hewlett-Packard 25 and the Texas Instruments SR-52 and SR-56 calculators.[59] By 1993 the attention Wolfe had previously paid to slide rules and hand-held calculators had been replaced with a number of BASIC programs to be run on a personal computer.[106] And so the radiation slide rule era had come to pass. No longer the calculational aid of choice, in a few short years it would all but disappear.

3.7 Scale summary

A comparison of the scales found on each of the various radiation slide rules discussed are listed in Table 1. Taken together the rules were capable of providing estimates for 30 different quantities, a staggering number considering the relative simplicity of the devices being used. Often on many of the rules more than one scale would be found for a particular quantity, wavelength being one common example. In addition to regular scales, gauge marks, multiplication scales, and suitable positioning of the cursor extended the capabilities of most rules by allowing additional quantities to be estimated. In counting the number of different physical quantities each rule could estimate, quantities involving maxima, such as wavelength and peak wavelength for example, were not considered to be different.

4. CONCLUSION

The inventiveness and dedication of a small number of people to design, develop and manufacture a modest variety of slide rules and slide charts for computation of a variety of radiometric blackbody quantities have positively impacted the entire infrared industry during its early growth years from the 1950s well into the 1980s. Untold thousands of scientists and engineers have utilized these remarkable tools and it was likely they keep these rules handy wherever they were working. Without question, the General Electric GEN-15C and the Infrared Industries, Inc. rules were the most prolifically distributed and used. The A. G. Thornton F5100, ARISTO, and Pickett Model 17 slide rules were extraordinary devices, beautifully constructed and highly functional, but are nowadays relatively rare. The Autonetics Photon Calculator is unique amongst the radiation slide rules in that it was circular and computed only actinometric quantities. It was also highly prized by those working with systems operating in ultra-low temperature environments. Examination of Table 1 shows the similarities and stark differences between the six basic slide rules and their variants described and illustrates how each developer had certain objectives and perhaps different philosophies in mind when designing each rule. Although it is evident that the era of the blackbody radiation slide rule has come to a graceful end, their contribution to the advancement of infrared technology has been significant and should never be forgotten.

ACKNOWLEDGMENTS

We are grateful to Prof. William L. Wolfe who kindly supplied the photograph of the now very rare ARISTO Nr. 10048 slide rule and to Mr Bruno Ferrighi for providing the photograph of the almost as rare ARISTO Nr. 922 rule. Thanks also to Mr Robert Adams for providing the two photographs of the Pickett Model 17 rule. We would also like to thank the following individuals for their recollections on using radiation slide rules: Prof. William L. Wolfe, Prof. Hans J. Queisser, Prof. Raymond J. Chandos, and Mr Max J. Riedl. These have been invaluable.

Table 1. A summary of the scales and gauge marks found on the various radiation slide rules described in the text.

	ARISTO		General Electric			Admiralty	Autonetics	IRI rule		EOI rule
	10048	922	GEN-15	GEN-15B	GEN-15C			cardboard	plastic[a]	
Temperature										
Kelvin	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Celsius	✗	✗	✓	✓	✓	✓	✗	✓	✓	✓
Fahrenheit	✗	✗	✓	✓	✓	✗	✗	✓	✗,✓	✓
Rankine	✗	✗	✓	✓	✓	✗	✗	✗	✗	✗
Radiometric and actinometric quantities										
$M^\dagger$	✗	✗	✓	✓	✓	✓	✗	✓	✓	✓
M_q	✗	✗	✗	✓	✓	✓	c	✓	✓	✓
$M_{q,0\to\lambda}$	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗
L	✓	✓	✗	g	g	✗	✗	✗	✗	✗
L_q	✗	✗	✗	✗	✗	✗	g	✗	✗	✗
E	✗	✗	✗	✓	✓	✗	✗	✗	✗	✗
$M_{\lambda_{\max}}$	✗	✗	✗	✓	✓	✓	✗	✓	✓	✓
$M_{q,\lambda_{\max}}$	✗	✗	✗	✗	✗	✓	✗	✓	✓	✓
emissivity[b]	✗	✗	✓	✓	✓	m	✗	✗	✗	m
Integrated and spectral ratios										
$\mathfrak{F}_{0\to\lambda}$	✗	✗	✓	✓	✓	✓	✗	✓	✓	✓
$\mathfrak{F}_{\lambda\to\infty}$	✓	✓	✗	✗	✗	✓	✗	✗	✗	✓
$\mathfrak{F}_{q,0\to\lambda}$	✗	✗	✗	✗	✗	✓	g	✓	✓	✓
$\mathfrak{F}_{q,\lambda\to\infty}$	✗	✗	✗	✗	✗	✓	✗	✗	✗	✓
$M_\lambda/M_{\lambda_{\max}}$	✗	✗	✗	✓	✓	✓	✗	✓	✓	✓
$M_{q,\lambda}/M_{q,\lambda_{\max}}$	✗	✗	✗	✗	✗	✓	✗	✓	✓	✓
Wavelength, wavenumber, and frequency										
λ	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\lambda_{\max}$	g	g	✗	g	g	g	✗	✗	✗	c
$\lambda_{q,\max}$	✗	✗	✗	✗	✗	g	g	✗	✗	c
$\bar{\nu}$	✗	✗	✓	✗	✗	✓	✗	✗	✗	✓
$\bar{\nu}_{\max}$	✗	✗	✗	✗	✗	g	✗	✗	✗	✗
$\bar{\nu}_{q,\max}$	✗	✗	✗	✗	✗	g	✗	✗	✗	✗
$\bar{\nu}_{\lambda_{\max}}$	✗	✗	✓	✗	✗	✗	✗	✓	✓	c
$\bar{\nu}_{q,\lambda_{\max}}$	✗	✗	✗	✗	✗	✗	✗	✗	✗	c
ν	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗
Other miscellaneous scales										
Photon energy	✗	✗	✓	✓	✓	✓	✗	✗	✗	✓
wavelength–temperature product, λT	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗
Transmission spectrum	✗	✗	✗	✓	✓	✗	✗	r	✗	✗
Relative luminosity	✗	✗	✓	✓	✓	✗	✗	✗	✗	✗
Johnson noise	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓
Range	✗	✗	✗	✓	✓	✗	✗	✗	✗	✗
Centimeters	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
C/D scales	✗	✗	✗	✓	✓	✗	✗	✗	✗	✓
TOTALS										
№ of scales	5	4	21	26	26	23	4	17	15,16	34
№ of gauge marks	2	2	7	5	5	4	2	0	0	2
№ of different physical quantities	4	4	9	13	13	14	3	11	11	13

[a] There are two different plastic versions. The first is for P1, the second for P2.

[b] The emissivity indicates it is possible to estimate from the rule either the total emittance, the spectral emittance at the spectral peak, or both, for a greybody.

† Scales for the total emittance were often given in more than one system of units.

Quantity found using: c – cursor, g – gauge mark, m – multiplication scale, r – reverse side of slide

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